BSR/ASHRAE Standard 41.7-2021R

Public Review Draft

Standard Methods for Gas Flow Measurement

First Public Review (April 2025)
(Complete Draft for Full Review)

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FOREWORD

This update of the 2021 version makes it easier for the higher-tier ASHRAE standards to adopt this standard by reference, updates the steady-state criteria requirements, and includes a new uncertainty example prepared in accordance with the latest uncertainty methods. This standard meets ASHRAE's mandatory language requirements.

Selecting an appropriate gas flowmeter can be a daunting task given the wide variety of operating principles, measurement precision, and costs of commercial products. Whether gas flow measurements are to be taken in a laboratory or in the field, selecting the appropriate meter should be based on the required measurement accuracy. Once a gas flowmeter has been selected, the user may need to consult with the meter manufacturer regarding installation specifics, operating range limits, calibration limits, and other similar specifics in order to obtain the expected measurement accuracy. Safety is an important consideration for all procedures involving gases, particularly regarding flammability, toxicity, and corrosiveness – wear safety glasses and other personal protection equipment.

1. PURPOSE

This standard prescribes methods for gas flow measurement.

2. SCOPE

This standard applies to laboratory and field gas flow measurement for testing heating, ventilating, air-conditioning, and refrigerating systems and components. This standard is restricted to applications where the entire flow stream of gas enters and exits the gas flowmeter in a "gas-only" state during data recording with the following exceptions:

a. This standard does not apply to airflow measurements at pressures within this range: -25 kPa to +25kPa (-100 in. of water to +100 in. of water) referenced to ambient pressure. Those measurements are within the scope of ANSI/ASHRAE Standard 41.2[1].

b. This standard does not apply to fan performance rating airflow measurements. Those measurements are within the scope of ANSI/ASHRAE Standard 51[2].

c. This standard does not apply to gaseous-phase refrigerant mass flow measurements where the gas flow includes circulating lubricant. Those measurements are within the scope of ANSI/ASHRAE Standard 41.10[3].

3. DEFINITIONS

The following definitions apply to the terms used in this standard.

***accuracy*:** the degree of conformity of an indicated value to the true value.

error: the difference between the test result and its corresponding true value.

geometrically equivalent diameter: the diameter of a circle having the same area as a non-circular gas flow area.

hydraulic diameter: four times the gas flow area divided by the perimeter of the solid boundary in contact with the gas.

measurement system: the instruments, signal conditioning systems if any, and data acquisition system if any.

operating tolerance limit: the upper or lower value of an operating tolerance that is associated with a test point or a targeted set point.

post-test uncertainty: an analysis to establish the uncertainty of a test result after conducting the test.

pretest uncertainty: an analysis to establish the expected uncertainty for a test result before conducting the test.

random error: the portion of the total error that varies randomly in repeated measurements of the true value throughout a test process.

steady-state criteria: the criteria that establish negligible change of gas flow with time.

systematic error: the portion of the total error that remains constant in repeated measurements of the true value throughout a test process.

targeted set point: a specific set of test conditions where the required gas mass flow rate is known and has an associated operating tolerance.

test point: a specific set of test operating conditions for recording data where the measured required gas mass flow rate is unknown and has an associated operating tolerance.

true value: the unknown, error-free value of a test result.

uncertainty: the limits of error within which the true value lies.

unit under test (UUT): equipment that is the subject of gas flow rate measurements.

4. CLASSIFICATIONS

4.1 Gas Flow Operating State. Gas flow measurement methods shall be restricted to applications where the entire gas flow stream enters and exits the gas flowmeter in the “vapor-only” state during data recording. Trace amounts of liquids shall be less than 1% by mass unless otherwise specified by the flowmeter manufacturer or by the Test Plan in Section 5.1.

4.2 Gas Flow Measurement Applications. Gas flow measurement applications that are within the scope of this standard shall be classified as one of the following types:

4.2.1 Laboratory Applications. Gas flow measurements under laboratory conditions are engineering development tests or tests to determine product ratings.

(***Informative Note:*** Laboratory gas flow measurements tend to use more accurate instruments than field measurements, and the installation of those instruments normally meets the instrument manufacturer’s installation requirements.)

4.2.2 Field Applications. Gas flow measurements under field conditions are tests to determine installed system gas flow rates.

(Informative Note: Field gas flow measurements tend to use less accurate instruments than laboratory measurements and often do not meet the instrument manufacturer's installation requirements.)

4.3 Gas Flow Meters

4.3.1 Gas Mass Flow Meters. Gas flow meters in this category perform direct measurement of gas mass flow rates.

4.3.2 Gas Volumetric Flow Meters. Gas flow meters in this category perform direct measurement of gas volumetric flow rates. If gas mass flow rates are required, each gas volumetric flow measurement shall be multiplied by the inlet gas density at the flow measurement location to obtain the gas mass flow rate measurement.

(Informative Note: Ultrasonic flowmeters and vortex-shedding flowmeters are examples of velocity-measuring devices that can be used to determine volumetric flow rates or gas mass flow rate if the density in the measurement plane is determinable.)

4.4 Gas Flow Measurement Methods. Gas flow measurement methods that are within the scope of this standard are the methods listed below. Each of these gas flow measurement methods is described in Section 7.5.

a. Coriolis flowmeters
b. Thermal flowmeters
c. Orifice meters
d. Flow nozzles
e. Venturi tubes
f. Turbine flowmeters
g. Variable-area flowmeters
h. Ultrasonic flowmeters
i. Pitot-static tube methods
j. Vortex-shedding flowmeters

5. REQUIREMENTS

5.1 Test Plan. A test plan shall specify the gaseous mass flow rate measurement system accuracy. The test plan shall also include the test points, targeted set points, and corresponding operating tolerances to be performed. The test plan shall be one of the following options:

a. A document provided by the person or the organization that authorized the tests and calculations to be performed.
b. A method of test standard.
c. A rating standard.
d. A regulation or code.
e. Any combination of items a. through d.

The test plan shall specify:

a. The minimum value for the accuracy or the maximum value of measurement uncertainty of the gas flow measurement system over the full range of operating conditions.
b. The values to be determined and recorded that are to be selected from this list: gas mass flow, pretest gas mass flow measurement uncertainty, post-test gas mass flow measurement uncertainty,

gas volumetric flow, pretest gas volumetric flow measurement uncertainty, and post-test gas flow measurement uncertainty.

c. Any combination of test points and targeted set points to be performed together with operating tolerances.

5.2 Values to be Determined and Reported. The test values to be determined and reported shall be as shown in Table 5-1. Use the unit of measure in the Table 5-1 unless otherwise specified in the test plan in Section 5.1.

Table 5-1 Measurement Values and Units of Measure

Quantity	SI	IP
Gas mass flow rate	kilogram per second (kg/s)	pound (avoirdupois) per minute (lb_m/min)
Gas mass flow rate uncertainty		
Gas volumetric flow rate	cubic meters per second (m^3/s)	cubic feet per minute (cfm)
Gas volumetric flow rate uncertainty		

5.3 Test Requirements

5.3.1 Pretest Uncertainty Analysis. If required by the test plan in Section 5.1, perform an analysis to establish the expected uncertainty for each gas mass flow or gas volumetric flow test point prior to the conduct of that test in accordance with the pretest uncertainty analysis procedures in ASME PTC 19.1[4].

5.3.2 Post-test Uncertainty Analysis. If required by the test plan in Section 5.1, perform an analysis to establish the expected uncertainty for each gas mass flow or gas volumetric test point in accordance with the post-test uncertainty analysis procedures in ASME PTC 19.1[4]. Alternatively, if specified in the test plan, the worst-case uncertainty for all test points shall be estimated and reported for each test point.

5.3.3 Steady-State Test Criteria for Gas Mass Flow Rate Measurements

5.3.3.1 Steady-State Test Criteria for Gas Mass Flow Rate Measurements Under Laboratory Test Conditions. If the test plan in Section 5.1 requires gas mass flow rate test data points to be recorded at steady-state test conditions and provides the operating condition tolerance but does not specify the steady-state criteria, then determine that steady-state test conditions have been achieved using one of the following methods:

a. Apply the steady-state criteria in Section 5.3.3.3_if the test plan provides test points for gas mass flow rate measurement.
b. Apply the steady-state criteria in Section 5.3.3.4_if the test plan provides targeted set points for gas mass flow rate measurement.

5.3.3.2 Steady-State Test Criteria for Gas Mass Flow Under Field Test Conditions. If the test plan in Section 5.1 requires test data points to be recorded at steady-state test conditions under field test conditions and provides the operating condition tolerance but does not specify the steady-state criteria, the methods in Section 5.3.3.1 are optional.

> (***Informative Note:*** The steady-state methods in Section 5.3.3.1 are likely to be impractical under field test conditions. Section 5.3.4 provides instructions for making measurements that are not at steady state conditions.)

5.3.3.3 Steady-State Gas Mass Flow Rate Criteria for Test Points

Starting with the time set to zero, sample not less than 30 gas mass flow rate measurements N at equal time intervals δt over a test duration Δt where Δt is in time units. Equation 5-1 states the relationship of the test duration to the number of gas mass flow rate samples and the equal time intervals.

$$\Delta t = (N - 1)\delta t \tag{5-1}$$

(Informative Note: Circumstances for measurement vary, so the user should select a duration of test and the equal time intervals based upon the longest period of the observed gas mass flow rate fluctuations during operation near the steady-state conditions.)

Record each sampled gas mass flow rate measurement $\dot{m}_i$ and the corresponding time t_i. Apply the least-squares line method to determine the slope b of the gas mass flow rate data trend using Equation 5-2.

$$b = \left\{ \frac{\left[N\left(\sum_{i=1}^{N} t_i \dot{m}_i\right) - \left(\sum_{i=1}^{N} t_i\right)\left(\sum_{i=1}^{N} \dot{m}_i\right)\right]}{\left[N\left(\sum_{i=1}^{N} {t_i}^2\right) - \left(\sum_{i=1}^{N} t_i\right)^2\right]} \right\} \tag{5-2}$$

(Informative Note: It should be noted that the units for the slope in Equation 5-2 are gas mass flow rate, kg/s (lb_m/min), divided by the units that the user has selected for time.)

The mean of the sampled gas mass flow rates $\bar{\dot{m}}$ is defined by Equation 5-3.

$$\bar{\dot{m}} = \frac{1}{N}\left[\sum_{i=1}^{N}(\dot{m}_i)\right], \text{ kg/s } (\text{lb}_\text{m}/\text{min}) \tag{5-3}$$

The difference between the maximum and minimum sampled values must be less than or equal to the specified test operating tolerance as defined in Equation 5-4 where $\dot{m}_L$ is the operating tolerance limit.

$$\dot{m}_{max} - \dot{m}_{imin} \leq \dot{m}_L \quad \text{kg/s } (\text{lb}_\text{m}/\text{h}) \tag{5-4}$$

The restriction on the slope of the trend line b is defined in Equation 5-5 where Δt is the sample time interval.

$$|b \times \Delta t| \leq 0.5 \times \dot{m}_L \quad \text{kg/s } (\text{lb}_\text{m}/\text{h}) \tag{5-5}$$

$\bar{\dot{m}}$, as determined by Equation 5-3, represents the steady-state mean gas mass flow rate where Equations 5-4 and 5-5 are both satisfied.

(Informative Note: For further reading about methods of determining steady-state conditions, refer to Informative Appendix A – Bibliography items A1 and A2.)

5.3.3.4 Steady-State Gas Mass Flow Rate Criteria for Targeted Set Points

Starting with the time set to zero, sample not less than 30 gas mass flow rate measurements N at equal time intervals δt over a test duration Δt where Δt is in time units. Equation 5-6 states the relationship of the test duration to the number of samples and the equal time intervals.

$$\Delta t = (N - 1)\delta t \tag{5-6}$$

(Informative Note: Circumstances for measurement vary, so the user should select a duration of test and the equal time intervals based upon the longest period of the observed gas mass flow rate fluctuations during operation near the steady-state conditions.)

Record each sampled gas mass flow rate measurement $\dot{m}_i$ and the corresponding time t_i. Apply the least-squares line method to determine the slope b of the gas mass flow rate data trend line using Equation 5-7.

$$b = \left\{ \frac{\left[N\left(\sum_{i=1}^{N} t_i \dot{m}_i\right) - \left(\sum_{i=1}^{N} t_i\right)\left(\sum_{i=1}^{N} \dot{m}_i\right)\right]}{\left[N\left(\sum_{i=1}^{N} {t_i}^2\right) - \left(\sum_{i=1}^{N} t_i\right)^2\right]} \right\} \tag{5-7}$$

*(**Informative Note:*** It should be noted that the units for the slope in Equation are gas mass flow rate, kg/s (lb_m/min), divided by the units that the user has selected for time.)

The mean of the sampled gas mass flow rates, $\bar{\dot{m}}$, is defined by Equation 5-8.

$$\bar{\dot{m}} = \frac{1}{N}\left[\sum_{i=1}^{N} (\dot{m}_i)\right], \text{ kg/s (lb}_\text{m}\text{/min)} \tag{5-8}$$

The difference between the maximum and minimum sampled values must be less than or equal to the specified test operating tolerance as defined in Equation 5-9 where $\dot{m}_L$ is the operating tolerance limit.

$$\dot{m}_{max} - \dot{m}_{min} \le \dot{m}_L \;\; \text{kg/s (lb}_\text{m}\text{/h)} \tag{5-9}$$

The restriction on the slope of the trend line b is defined in Equation 5-10 where Δt is the sample time interval.

$$|b \times \Delta t| \le 0.5 \times \dot{m}_L \;\; \text{kg/s (lb}_\text{m}\text{/h)} \tag{5-10}$$

The difference between the test condition and mean of the sampled values shall be less than or equal to half of the specified operating tolerance limit as defined in Equation 5-11 where $\dot{m}_{SP}$ is the set point mass flow rate and $\dot{m}_L$ is the operating tolerance limit.

$$|\dot{m}_{SP} - \bar{\dot{m}}| \le 0.5 \times \dot{m}_L \;\; \text{kg/s (lb}_\text{m}\text{/h)} \tag{5-11}$$

$\bar{\dot{m}}$, as determined by Equation 5-8, represents the steady-state mean gas mass flow rate where Equations 5-9, 5-10, and 5-11 are all satisfied.

*(**Informative Note:*** For further reading about methods of determining steady-state conditions, refer to Informative Appendix A – Bibliography items A1 and A2.)

5.3.4 Unsteady Gas Mass Flow Rate Measurements. If required by the test plan in Section 5.1, gas mass flow rate test data shall be recorded:

a. at operating conditions that are not steady state,
b. at the time intervals specified in the test plan,
c. within the test condition limits specified in the test plan,
d. using instrument response times specified in the test plan.

5.3.5 Steady-State Test Criteria for Gas Volumetric Flow Rate Measurements

5.3.5.1 Steady-State Test Criteria for Gas Volumetric Flow Rate Measurements Under Laboratory Test Conditions. If the test plan requires gas volumetric flow rate test data points to be recorded at steady-state test conditions and provides the operating condition tolerance but does not specify the steady-state criteria, then determine that steady-state test conditions have been achieved using one of the following methods:

a. Apply the steady-state criteria in Section 5.3.6.3 if the test plan provides test points for gas volumetric flow rate measurement.

b. Apply the steady-state criteria in Section 5.3.6.4 if the test plan provides targeted set points for gas volumetric flow rate measurement.

5.3.5.2 Steady-State Test Criteria for Gas Volumetric Flow Under Field Test Conditions. If the test plan in Section 5.1 requires test data points to be recorded at steady-state test conditions under field test conditions and provides the operating condition tolerance but does not specify the steady-state criteria, the methods in Section 5.3.5.1 are optional.

> ***(Informative Note:*** The steady-state methods in Section 5.3.5.1 are likely to be impractical under field test conditions. Section 5.3.6 provides instructions for making measurements that are not at steady state conditions.)

5.3.5.3 Steady-State Gas Volumetric Flow Rate Criteria for Test Points

Starting with the time set to zero, sample not less than 30 gas volumetric flow rate measurements N at equal time intervals δt over a test duration Δt where Δt is in time units. Equation 5-12 states the relationship of the test duration to the number of gas volumetric flow rate samples and the equal time intervals.

$$\Delta t = (N-1)\delta t \qquad (5\text{-}12)$$

> ***(Informative Note:*** Circumstances for measurement vary, so the user should select a duration of test and the equal time intervals based upon the longest period of the observed gas volumetric flow rate fluctuations during operation near the steady-state conditions.)

Record each sampled gas volumetric flow rate measurement Q_i and the corresponding time t_i. Apply the least-squares line method to determine the slope b of the gas volumetric flow rate data trend line using Equation 5-13.

$$b = \left\{ \frac{\left[N\left(\sum_{i=1}^{N} t_i Q_i\right) - \left(\sum_{i=1}^{N} t_i\right)\left(\sum_{i=1}^{N} Q_i\right)\right]}{\left[N\left(\sum_{i=1}^{N} {t_i}^2\right) - \left(\sum_{i=1}^{N} t_i\right)^2\right]} \right\} \qquad (5\text{-}13)$$

> ***(Informative Note:*** It should be noted that the units for the slope in Equation 5-13 are gas volumetric flow rate, m^3/s (cfm), divided by the units that the user has selected for time.)

The mean of the sampled gas volumetric flow rates $\bar{Q}$ is defined by Equation 5-14.

$$\bar{Q} = \frac{1}{N}\left[\sum_{i=1}^{N}(Q_i)\right], \text{ m}^3\text{/s (cfm)} \qquad (5\text{-}14)$$

The difference between the maximum and minimum sampled values shall be less than or equal to the specified operating tolerance limit as defined in Equation 5-15 where Q_L is the operating tolerance limit.

$$Q_{max} - Q_{imin} \leq \mathrm{Q}_L \text{ m}^3\text{/s (cfm)} \qquad (5\text{-}15)$$

The restriction on the slope of the trend line b is defined in Equation 5-16 where Δt is the sample time interval.

$$|b \times \Delta t| \leq 0.5 \times \mathrm{Q}_L \text{ m}^3\text{/s (cfm)} \qquad (5\text{-}16)$$

$\bar{Q}$, as determined by Equation 5-14, represents the steady-state mean gas volumetric flow rate where Equations 5-15 and 5-16 are both satisfied.

> ***(Informative Note:*** For further reading about methods of determining steady-state conditions, refer

to Informative Appendix A – Bibliography items A1 and A2.)

5.3.5.4 Steady-State Gas Volumetric Flow Rate Criteria for Targeted Set Points
Starting with the time set to zero, sample not less than 30 gas volumetric flow rate measurements N at equal time intervals δt over a test duration Δt where Δt is in time units. Equation 5-17 states the relationship of the test duration to the number of samples and the equal time intervals.

$$\Delta t = (N-1)\delta t \tag{5-17}$$

(Informative Note: Circumstances for measurement vary, so the user should select a duration of test and the equal time intervals based upon the longest period of the observed gas volumetric flow rate fluctuations during operation near the steady-state conditions.)

Record each sampled gas volumetric flow rate measurement Q_i and the corresponding time t_i. Apply the least-squares line method to determine the slope b of the gas volumetric flow rate data trend line using Equation 5-18.

$$b = \left\{ \frac{\left[N\left(\sum_{i=1}^{N} t_i Q_i\right) - \left(\sum_{i=1}^{N} t_i\right)\left(\sum_{i=1}^{N} Q_i\right)\right]}{\left[N\left(\sum_{i=1}^{N} {t_i}^2\right) - \left(\sum_{i=1}^{N} t_i\right)^2\right]} \right\} \tag{5-18}$$

(Informative Note: It should be noted that the units for the slope in Equation 5-18 are gas volumetric flow rate, m^3/s (cfm), divided by the units that the user has selected for time.)

The mean of the sampled gas volumetric flow rates $\bar{Q}$ is defined by Equation 5-19.

$$\bar{Q} = \frac{1}{N}\left[\sum_{i=1}^{N}(Q_i)\right], \text{ m}^3\text{/s (cfm)} \tag{5-19}$$

The difference between the maximum and minimum sampled values shall be less than or equal to the specified operating tolerance limit as defined in Equation 5-20 where Q_L is the operating tolerance limit.

$$Q_{max} - Q_{min} \le \mathrm{Q}_L \text{ m}^3\text{/s (cfm)} \tag{5-20}$$

The restriction on the slope of the trend line b is defined in Equation 5-21 where Δt is the sample time interval.

$$|b \times \Delta t| \le 0.5 \times Q_L \text{ m}^3\text{/s (cfm)} \tag{5-21}$$

The difference between the test condition and mean of the sampled values shall be less than or equal to half of the specified operating tolerance limit as defined in Equation 5-22 where Q_{SP} is the set point volumetric flow rate and Q_L is the operating tolerance limit.

$$|Q_{SP} - \bar{Q}| \le 0.5 \times \mathrm{Q}_L \text{ m}^3\text{/s (cfm)} \tag{5-21}$$

$\bar{Q}$, as determined by Equation 5-19, represents the steady-state mean gas volumetric flow rate where Equations 5-20, 5-21, and 5-22 are all satisfied.

(Informative Note: For further reading about methods of determining steady-state conditions, refer to Informative Appendix A – Bibliography items A1 and A2.)

5.3.6 Unsteady Gas Volumetric Flow Rate Measurements. If required by the test plan in Section 5.1, gas mass flow rate test data or gas volumetric flow rate data shall be recorded:

1. at operating conditions that are not steady state,

2. at the time intervals specified in the test plan,
3. within the test condition limits specified in the test plan,
4. using instrument response times specified in the test plan.

5.3.7 Gas Properties. If not specified in the test plan in Section 5.1, the gas property data shall be obtained from the NIST Standard Reference Database 23 (REFPROP)[5] or from the source of the gas and shall be recorded in the test report.

5.3.8 Operating Limits. Operating conditions during gas flow rate data measurements shall not exceed limits for pressure, pressure differential, temperature, gas velocity, or pressure pulsations specified in the test plan or by the gas flowmeter manufacturer to achieve the measurement system accuracy required by the test plan.

5.3.9 Leakage Requirement. Unless otherwise specified in the test plan in Section 5.1, measured gas leakage out of the test apparatus shall be not be greater than 0.25% of the gas flow rate at the greatest pressure tested under laboratory conditions, or not greater than 1% of the gas flow rate at the greatest pressure tested under field conditions.

(Informative Note: Account for the leakage in the uncertainty analysis.)

5.3.10 Gas Flowmeter Installation. The selected gas flowmeter shall be installed in accordance with instructions from the manufacturer, or the uncertainty calculations shall include estimated uncertainties for installations that are not in accordance with the manufacturer's instructions.

6. INSTRUMENTS

6.1 Instrumentation Requirements for All Measurements

6.1.1 Instruments and data acquisition systems shall be selected to meet the measurement system accuracy _ specified in the test plan.

6.1.2 Measurements from the instruments shall be traceable to primary or secondary standards calibrated by the National Institute of Standards and Technology (NIST) or to the Bureau International des Poids et Mesures (BIPM) if a National Metrology Institute (NMI) other than NIST is used. In either case, the indicated corrections shall be applied to meet the uncertainty stated in subsequent sections. Instruments shall be recalibrated on regular intervals that do not exceed the intervals prescribed by the instrument manufacturer and calibration records shall be maintained. Instruments shall be installed in accordance with the instrument manufacturer's requirements or the manufacturer's accuracy does not apply.

6.1.3 Instruments shall be applied and used in accordance with the following standards:

a. Temperature – ANSI/ASHRAE Standard 41.1[6] if temperature measurements are required.
b. Pressure – ANSI/ASHRAE Standard 41.3[7] if pressure measurements are required.

6.2 Temperature Measurements

If temperature measurements are required by the test plan in Section 5.1, the measurement system accuracy shall be within the following limits unless otherwise specified in the test plan:

a. Temperature sensors within ±0.28°C (±0.5°F).
b. Temperature difference sensors within ±l.0% of the reading.

6.3 Pressure Measurements

6.3.1 Laboratory Pressure Measurements

6.3.1.1 If pressure measurements are required by the test plan in Section 5.1, the measurement system accuracy shall be within ±1.0% of reading unless otherwise specified in the test plan. If absolute pressure sensors are not used, the barometric pressure shall be added to obtain absolute pressure values prior to performing uncertainty calculations.

6.3.1.2 If differential pressure measurements are required by the test plan, the measurement system accuracy shall be within ±1.0% of reading unless otherwise specified in the test plan. Pressure shall be measured in close proximity to the flow meter in accordance with the flow meter manufacturer's specifications.

6.3.2 Field Pressure Measurements

6.3.2.1 If pressure measurements are required by the test plan in Section 5.1, the measurement system accuracy shall be within ±3.0% of reading unless otherwise specified in the test plan. If absolute pressure sensors are not used, the barometric pressure shall be added to obtain absolute pressure values prior to performing uncertainty calculations.

6.3.2.2 If differential pressure measurements are required by the test plan, the measurement system accuracy shall be within ±3.0% of reading unless otherwise specified in the test plan. Pressure shall be measured in close proximity to the flow meter in accordance with the flow meter manufacturer's specifications.

6.4. Time Measurements

Time measurement system accuracy shall be within ±0.5% of the elapsed time measured, including any uncertainty associated with starting and stopping the time measurement unless (a) otherwise specified in the test plan in Section 5.1, or (b) a different value for time measurement system accuracy is required to be consistent with the gas flow rate measurement system accuracy specified in the test plan.

7. GAS FLOW RATE MEASUREMENT METHODS

7.1 Constraint on All Gas Flow Rate Measurement Methods. A selected gas flow measurement plane shall exceed 7.5 geometrically equivalent diameters downstream of an obstruction or any change in the gas flow direction and shall exceed 3 geometrically equivalent diameters upstream of an obstruction or change in the gas flow direction unless otherwise specified by the gas flow measurement instrument manufacturer. For a non-circular duct with a gas flow area *A,* the geometrically equivalent diameter shall be obtained from Equation 7-1. For a round duct, the geometrically equivalent diameter D_E is equal to the interior diameter *D.*

$$D_E = \sqrt{\frac{4A}{\pi}} \tag{7-1}$$

where
D_E = geometrically equivalent diameter, m (ft)
A = gas flow area, m^2 (ft^2)

7.2 Coriolis Flowmeters. Review Section 7.1. Coriolis gas flowmeters provide direct measurement of gas mass flow rates. In a Coriolis flowmeter, the gas flows through a vibrating sensor tube within the meter. An electromagnetic coil located on the sensor tube vibrates the tube in a cantilever motion at a known frequency. The gas enters a vibrating tube and is given the vertical momentum of the tube. The gas in the entry portion of the sensor tube resists in the downward direction when the tube is moving upward during half of the vibration cycle. Conversely, when the tube is moving downward during half of the vibration cycle, the gas in the exit portion of the sensor tube resists in the upward direction. Combined, these effects create a symmetrical twist angle. According to Newton's Second Law of Motion, the amount of sensor tube twist angle is directly proportional to the mass flow rate of gas flowing through the tube. Electromagnetic velocity sensors, located on opposing sides of the sensor tube, measure the velocity of the vibrating tube. Mass flow rate is determined by measuring the time difference in the velocity measurements – the greater the time difference, the greater the mass flow rate.

(Informative Note: For further reading, see Informative Appendix A Section A3.)

7.3 Thermal Flowmeters. Review Section 7.1. Thermal flowmeters provide direct measurement of gas mass flow rates. The basic elements of the constant heat input thermal mass flowmeters are two temperature sensors that are positioned on opposite sides of an electric heater. The gas mass flow rate shall be obtained from Equation 7-2.

(Informative Note: For further reading, see Informative Appendix A Section A4.)

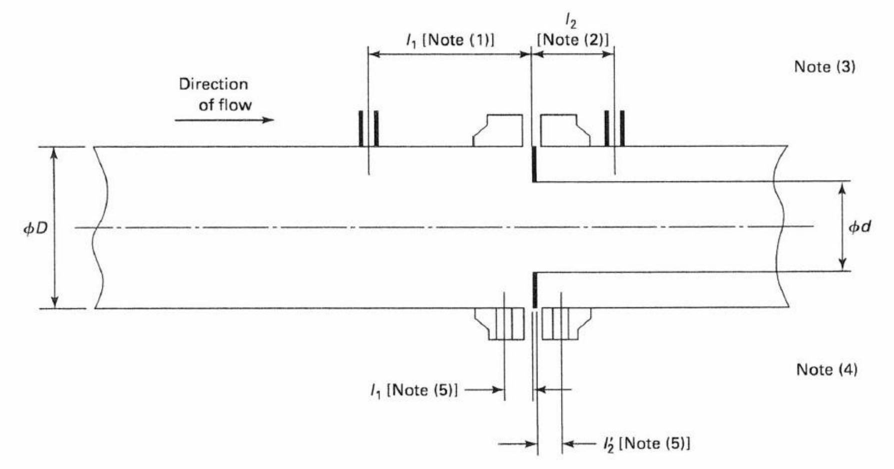
$$\dot{m} = \frac{Kq}{C_p(T_2 - T_1)} \tag{7-2}$$

where

$\dot{m}$ = gas mass flow rate, kg/s (lb_m/min)
K = dimensionless meter coefficient
q = electric heat flux rate, kJ/s (Btu/min)
C_p = specific heat of the gas, kJ/kg-K ~~(Btu/lb-°F)~~ (Btu/lb_m-°F)
T_1 = incoming gas temperature, °C (°F)
T_2 = outgoing gas temperature, °C (°F)

7.4 Orifices, Flow Nozzles, and Venturi Tube Flow Meters. Review Section 7.1. Orifices, flow nozzles, and venturi tubes are mass flow meters. ASME PTC 19.5[5] and ASME MFC-3M[6] describe measurement of fluid flow in pipes using orifices, flow nozzles, and venturi tubes, including construction proportions and port locations.

7.4.1 Orifices, Flow Nozzles, and Venturi Tube Flowmeter Geometric Profiles. Figure 7-1 illustrates the geometric profile of an orifice metering section. Figure 7-2 illustrates the geometric profile of a long radius nozzle, and Figure 7-3 shows the geometric profile of a venturi tube.



NOTES:
(1) $l_1 = D \pm 0.1D$.
(2) $l_2 = 0.5D \pm 0.02D$ for $\beta \leq 0.6$.
$= 0.5D \pm 0.01D$ for $\beta > 0.6$.
(3) D and $D/2$ pressure tap arrangement.
(4) Flange tap arrangement.
(5) $l_1 = l_2' = 25.4$ mm ± 0.5 mm (1.00 in. ± 0.02 in.) for $\beta > 0.6$ and $D < 150$ mm (6 in.)
$= 25.4$ mm ± 1 mm (1.00 in. ± 0.04 in.) for $\beta \leq 0.6$
$= 25.4$ mm ± 1 mm (1.00 in. ± 0.04 in.) for $\beta > 0.6$ and 150 mm $\leq D \leq$ 1000 mm (6 in. $\leq D \leq$ 40 in.)

Figure 7-1: Orifice Flowmeter Geometric Profile
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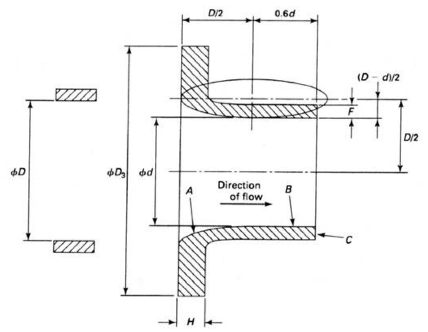
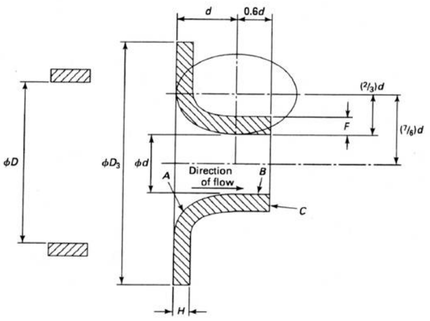


(a) High Ratio $0.25 \leq \beta \leq 0.8$



(b) Low Ratio $0.2 \leq \beta \leq 0.5$

Figure 7-2: Long Radius Nozzle Geometric Profile
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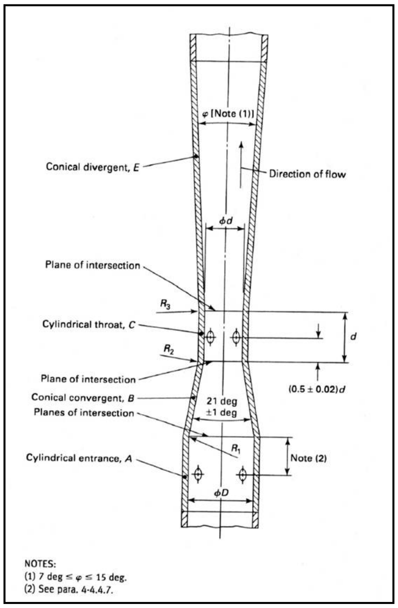

Figure 7-3: Venturi Tube Flowmeter Geometric Profile
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7.4.2 Gas Mass Flow Rate Equations and Procedures. This section provides the equations procedures for calculating gas mass flow rates using long radius nozzles and provides reference information for calculating gas mass flow rates for orifices, ISA 1932 nozzles, venturi nozzles, or venturi tubes.

Calculating a gas mass flow rate using these methods requires iteration because (a) the discharge coefficient *C* is a function of the Reynolds number and the Reynolds number is a function of the average gas flow velocity, and (b) the average gas flow velocity is not known until the gas mass flow rate has been determined. ASME PTC 19.5[8] includes an example of this iterative procedure on page 25, and ASME MFC-3M[9] provides the limits of use, discharge coefficient equations, and expansibility factor equations for orifices, long radius nozzles, ISA 1932 nozzles, venturi nozzles, and venturi tube flowmeters.

7.4.2.1 Measurements: Measurements required for this nozzle gas flow shall be:

a. Inlet duct geometrically equivalent diameter D_E that is defined in Equation 7-1, m (ft)
b. Nozzle throat diameter d, m (ft)
c. Nozzle inlet absolute pressure p_1, Pa (in. of water)
d. Nozzle throat absolute pressure p_2, Pa (in. of water)
e. Nozzle differential pressure $\Delta p = (p_1 - p_2)$, Pa (in. of water)
f. Nozzle inlet temperature t_1, °C (°F)

7.4.2.2 Nozzle Inlet Duct Hydraulic Diameter. Nozzle inlet duct hydraulic diameter D_h shall be obtained from dimensional measurements. For a nozzle inlet duct with gas flow area A, the hydraulic diameter shall be obtained from Equation 7-3. For a round duct D_h is equal to the interior inlet diameter.

$$D_h = \frac{4A}{P_{wetted}} \tag{7-3}$$

where
D_h = hydraulic diameter, dimensionless
A = gas flow area m^2 (ft^2)
P_{wetted} = nozzle duct perimeter that is in contact with the gas flow, m (ft)

7.4.2.3 Nozzle Limits for Use and Reynolds Number. Limits for the use for long radius nozzle are:

a. 50 mm (2 in.) $\leq D \leq$ 630 mm (25 in.)
b. $R_a/D \leq 3.2\ (10^{-4})$ where R_a is the mean of the surface roughness in the upstream duct
c. $1\ (10^4) \leq Re_D \leq 1000\ (10^7)$ where Re_D is defined in Equation 7-4.

$$Re_D = \frac{\rho_1 V D_h}{\mu} \tag{7-4}$$

where
ρ_1 = gas density, kg/m^3 (lb$_m$/ft^3)
V = average gas velocity $\left[\frac{\dot{m}}{\rho_1 A}\right]$, m/s (ft/s)
D_h = nozzle inlet hydraulic diameter, m (ft)
μ = dynamic viscosity, Ns/m^2 (lb$_m$/s-ft)

7.4.2.4 Nozzle Beta Ratio. The nozzle beta ratio shall be obtained from Equation 7-5. If gas flow operating temperatures are not within ±6ºC (±10ºF) of the ambient temperature during the dimensional measurements, parameters d, D_h, and β shall be corrected to account for thermal expansion in compliance with ASME PTC 19.5[7] Section 3-10.

$$\beta = \left(\frac{d}{D_h}\right), \text{ dimensionless} \tag{7-5}$$

7.4.2.5 Nozzle Inlet Gas Density. The nozzle inlet gas density ρ_1 shall be obtained from the gas property data prescribed in Section 5.3.8 as a function of the nozzle inlet temperature t_1 and pressure p_1 at each data point.

7.4.2.6 Nozzle Gas Volumetric Flow Rates. Nozzle gas volumetric flow rates shall be calculated from Equation 7-6 in SI units or Equation 7-7 in I-P units.

In SI units:

$$Q = C\varepsilon\left(\frac{\pi}{4}\right) d^2 K_1 \sqrt{\frac{2(\Delta p)}{\rho_1(1-E\beta^4)}} \tag{7-6}$$

where

Q = nozzle gas volumetric flow rate, m^3/s
C = nozzle discharge coefficient, dimensionless
ε = nozzle expansibility factor, dimensionless
d = nozzle throat diameter, m
K_1 = nozzle calibration coefficient, dimensionless
ρ_1 = nozzle inlet gas density, kg/m^3
Δp = nozzle differential pressure, Pa
E = flow kinetic energy coefficient = 1.043[6]
β = d/D_h, dimensionless

In I-P units:

$$Q = 1097.8 \times C\varepsilon\left(\frac{\pi}{4}\right) d^2 K_1 \sqrt{\frac{(\Delta p)}{\rho_1(1-E\beta^4)}} \tag{7-7}$$

where

Q = nozzle gas volumetric flow rate, cfm
C = nozzle discharge coefficient, dimensionless
ε = nozzle expansibility factor, dimensionless
d = nozzle throat diameter, ft
$\underline{K_1}$ = nozzle calibration coefficient, dimensionless
ρ_1 = nozzle inlet gas density, $\text{lb}_\text{m}/\text{ft}^3$
Δp = nozzle differential pressure, in. of water
E = flow kinetic energy coefficient = 1.043[6]
β = d/D_h, dimensionless

7.4.2.7 Nozzle Expansibility Factor. The dimensionless nozzle expansibility factor ε for a long radius nozzle shall be obtained from Equation 7-8. This equation assumes that the gas is an ideal gas and the gas flow is an isentropic process.

$$\varepsilon = \left[r^{\frac{2}{\gamma}} \left(\frac{\gamma}{\gamma-1}\right) \left(\frac{1-r^{\frac{\gamma-1}{\gamma}}}{1-r}\right) \left(\frac{1-\beta^4}{1-\beta^4 r^{\frac{2}{\gamma}}}\right) \right]^{1/2} \tag{7-8}$$

where

r = absolute pressure ratio $\left[\frac{p_2}{p_1}\right]$, dimensionless
γ = ratio of specific heats $\left[\frac{C_p}{C_v}\right]$, dimensionless
β = d/D_h, dimensionless

7.4.2.8 Nozzle Discharge Coefficient. The dimensionless nozzle discharge coefficient C is a function of β and the Reynolds number based upon the nozzle inlet diameter. The discharge coefficient C for long radius nozzles shall be obtained from Equation 7-9.

$$C = 0.9965 - (0.00653\beta^{0.5})\left(\frac{10^6}{Re_D}\right)^{0.5} \tag{7-9}$$

The Reynolds number shall be calculated from 7-3, but the average velocity is not known until the gas mass flow rate has been determined. Iteration is required to determine the gas mass flow rate. Choose C = 1.0 to begin the iterative calculation procedure for long radius nozzles, ISA 1932 nozzles, venturi nozzles and for venturi tube flowmeters, or choose C = 0.6 for orifice flowmeters. Iteration shall continue until the calculated discharge coefficient C matches the previous discharge coefficient within ±0.005. To calculate gas mass flow rates for orifices, ISA 1932 nozzles, venturi nozzles, or venturi tubes, refer to the paragraphs in ASME MFC-3M[5] that are listed in Table 7-1 and use the same procedures that have been described for the long radius nozzles.

Table 7-1 References in ASME MFC-3M[6] for ISA 1932 Nozzles, Venturi Nozzles, and Venturi Tubes

Flowmeter Type	Limit of Use Section Number	Discharge Coefficient Equation	Expansibility Factor Equation
Orifices	2-4.3.1	2-4	2-6
ISA 1932 Nozzles	3-4.1.6.1	3-6	3-7
Venturi Nozzles	3-4.3.4.1	3-16	3-7
Venturi Tubes	4-4.5.1	4-4.5.1, 4-5.4.2, or 4-4.5.3	4-3

7.4.2.9 Nozzle Gas Mass Flow Rate. The nozzle gas mass flow rate shall be obtained from Equation 7-10, where ρ_1 is the nozzle inlet gas density, kg/m^3 (lb$_m$/ft^3) and Q is the gas volumetric flow rate, m^3/s (cfm), using Equation 7-5 in SI units or Equation 7-6 in I-P units.

$$\dot{m} = \rho_1 Q, \text{ kg/s (lb}_m\text{/min)} \qquad (7\text{-}10)$$

7.5 Turbine Flow Meters. Review Section 7.1. Turbine flowmeters are volumetric flow meters that have a turbine rotor suspended on low-friction bearings in the gas stream. The rotational speed of the turbine is a linear function of the average gas velocity, and is therefore a linear function of the volumetric flow rate. Turbine rotation is sensed by one of these methods: (a) reluctance sensors, (b) inductance sensors, (c) capacitance sensors, (d) Hall-effect sensors, or (e) mechanical sensors.

(Informative Note: For further reading, see Informative Appendix A Section A5.)

7.6 Variable-Area Flowmeters. Review Section 7.1. Variable-area flowmeters are volumetric flowmeters. These flowmeters consist of a float that is free to move vertically inside a tapered transparent tube that has a graduated scale as shown in Figure 7-4. The gas to be metered enters at the narrow bottom end of the tube and moves upward, passing through the annulus formed between the float and the inside wall of the tube. The position of the float is a balance between the gas pressure forces across the annulus acting upward and gravity acting downward on the float.

(Informative Note: For further reading, see Informative Appendix A Section A6.)

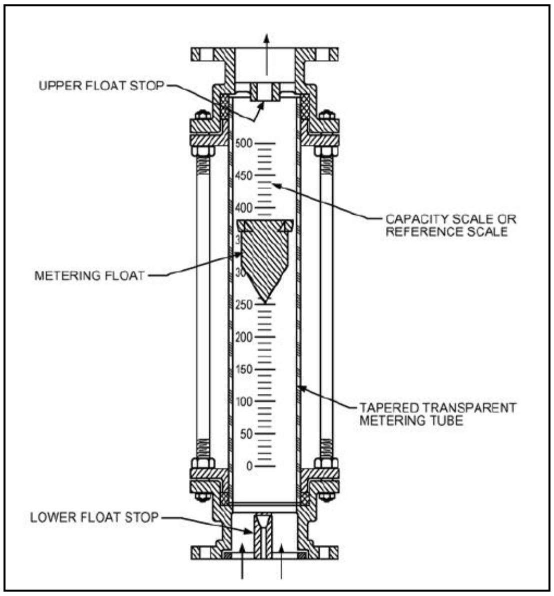


Figure 7-4: Variable-Area Flowmeter

7.7 Ultrasonic Flowmeters. Review Section 7.1. Ultrasonic flowmeters measure gas flow velocity. Clamp-on ultrasonic flow meters measure gas velocity within a pipe or tube without being inserted into the flow stream.

(Informative Notes:

a. Immersion-type ultrasonic flowmeters are also available.
b. For further reading, see Informative Appendix A Section A4.
c. Some ultrasonic flowmeters report a volumetric flow rate that is based on a specific pipe diameter.)

Ultrasonic flowmeters use the transit-time method to measure the effects that flow velocity has on bi-directional acoustical signals. An upstream transducer sends a signal to a downstream transducer that then returns a signal. When there is no flow, the time for the signal to go from one transducer to other, in either direction, is constant. When gas flow exists, the velocity causes the acoustical signal to increase speed in the direction of flow and reduces the acoustical signal speed in the upstream direction. This creates the time difference that correlates to the flow velocity.

7.8 Pitot-Static Tube Gas Flow Measurement Methods. Review Section 7.1. Figure 7-5 shows an example Pitot-static tube construction and the connections to manometers or electronic pressure transducers. Sections 7.8.1, 7.8.2, and 7.8.3 describe three different methods to determine gas velocity at measurement points in a gas stream by measuring total and static pressures. Pitot-static tubes shall be aligned within ±10 degrees of the gas flow direction, and any misalignment shall be included in the uncertainty estimate.

(Informative Notes:

a. Negative pressure readings are a clear indication that the Pitot-static tube is not properly aligned with the direction of gas flow.
b. The presence of significant thermal gradients leads to significant reductions in accuracies unless temperature is measured at each measurement location.)

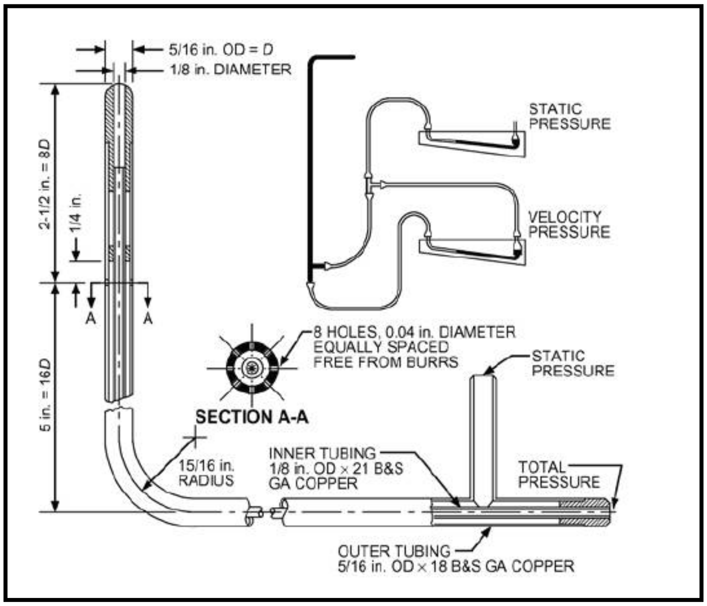
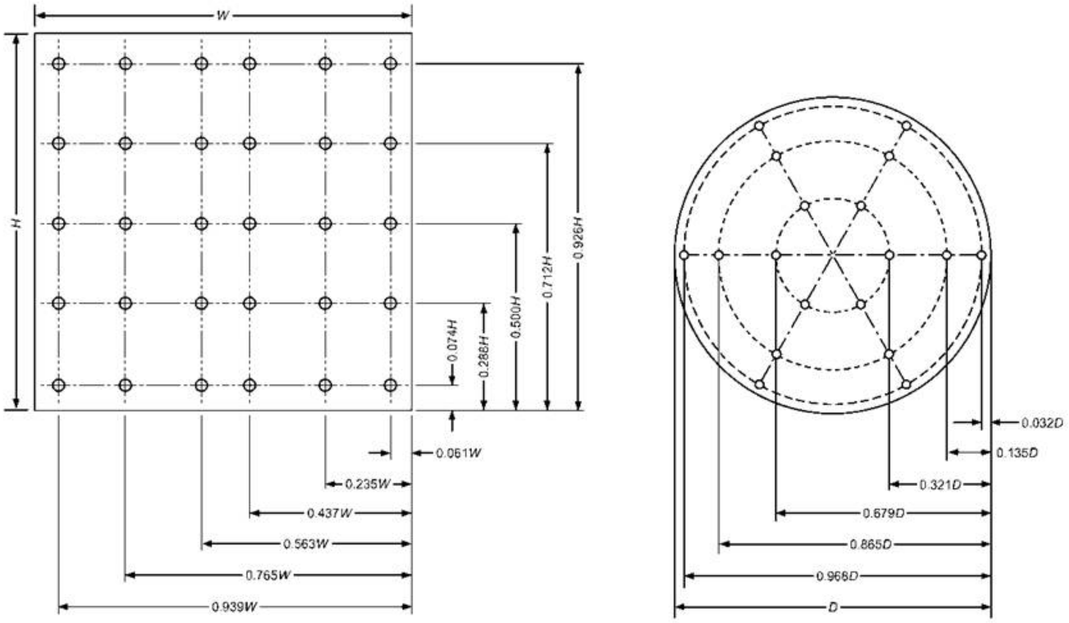


FIGURE 7-5: An Example of a Pitot-Static Tube

7.8.1 Pitot-Static Tube Traverse Gas Flow Measurement. The process of sequentially positioning a Pitot-static tube at different measuring points within a duct cross section to measure gas velocities is called a Pitot-static tube traverse. The traverse measuring points for a round duct or rectangular duct shall be in accordance with Figure 7-6.



Duct Dimensions	No. of Points for Traverse Lines	Position Relative to Inner Wall
460 mm < *H*, *W* < 750 mm	5	0.074. 0.288, 0.500, 0.712, 0.926
750 mm ≤ *H*, *W* ≤ 900 mm	6	0.061. 0.235, 0.437, 0.563, 0.765. 0.939
H, *W* > 900 mm	7	0.053. 0.203, 0.366, 0.500, 0.634, 0.797, 0.947

Log-Tchebycheff Rule for Rectangular Ducts

No. of Measuring Points per Diameter	Position Relative to Inner Wall
6	0.032, 0.135. 0.321, 0.679, 0.865, 0.968
8	0.021, 0.117, 0.184, 0.345, 0.655, 0.816, 0.883, 0.979
10	0.019, 0.077, 0.153, 0.217, 0.361, 0.639 0.783, 0.847, 0.923, 0.981

Log-Linear Rule for Circular Ducts

FIGURE 7-6. Pitot-Tube Traverse Measuring Points for Rectangular Ducts and Round Ducts

7.8.1.1 Velocity Pressure. The total pressure P_t is the sum of the static pressure P_s, and the velocity pressure P_v so it follows that

$$P_v = P_t - P_s, \text{ Pa (in. of water)} \qquad (7\text{-}11)$$

7.8.1.2 Average Velocity Pressure. The average velocity pressure P_{va} shall be obtained from Equation 7-12 where N is the number of velocity pressure sampling points

$$P_{va} = \left(\frac{\sum_{i=1}^{N} \sqrt{P_{vi}}}{N}\right)^2, \text{ Pa (in. of water)} \qquad (7\text{-}12)$$

7.8.1.3 Average Gas Velocity. The average gas velocity shall be obtained from the density at the traverse plane and the average velocity pressure from Equation 7-13 in SI units and from Equation 7-14 in I-P units.

In SI units:

$$V_a = K_2 \sqrt{\frac{2P_{va}}{\rho}} \qquad (7\text{-}13)$$

where

V_a = average gas velocity, m/s
K_2 = calibration coefficient, dimensionless
P_{va} = pressure, Pa
ρ = gas density in the measurement plane, kg/m^3

In I-P units:

$$V_a = 1097.8K_2\sqrt{\frac{P_{va}}{\rho}} \tag{7-14}$$

where

V_a = average gas velocity, ft/min
K_2 = calibration coefficient, dimensionless
P_{va} = pressure, in. of water
ρ = gas density in the measurement plane, lb$_m$/ft^3

7.8.1.4 Gas Volumetric Flow. The gas volumetric flow at the Pitot-static tube traverse plane shall be obtained from Equation 7-15.

$$Q = V_a A \tag{7-15}$$

where

Q = gas volumetric flow rate, m^3/s (cfm)
V_a = average gas velocity, m/s (ft/min)
A = measurement plane cross section area, m^2 (ft^2)

7.8.2 Self-Averaging Array Gas Flow Measurement. Self-averaging arrays consist of multiple bifurcated or extruded tubes spread out over a measurement plane that have holes to sample and self-average both total and static pressure across the measurement plane. The self-averaged total pressure is connected to one side of a differential pressure transducer, and the self-averaged static pressure is connected to the other side of the same pressure transducer.

7.8.2.1 Average Velocity Pressure. The average velocity pressure shall be obtained from Equation 7-16.

$$P_{va} = P_{ta} - P_{sa} \tag{7-16}$$

where

P_{va} = average velocity pressure, Pa (in. of water)
P_{ta} = measured average total pressure, Pa (in. of water)
P_{sa} = measured average static pressure, Pa (in. of water)

7.8.2.2 Average Gas Velocity. The average gas velocity shall be obtained from Equation 7-17 in SI units or from Equation 7-18 in I-P units:

In SI units:

$$V_a = K_3\sqrt{\frac{2P_{va}}{\rho}} \tag{7-17}$$

where

V_a = average gas velocity, m/s
K_3 = calibration coefficient, dimensionless
P_{va} = pressure, Pa
ρ = gas density in the measurement plane, kg/m^3

In I-P units:

$$V_a = 1097.8K_3\sqrt{\frac{P_{va}}{\rho}} \tag{7-18}$$

where

V_a = average gas velocity, ft/min
K_3 = calibration coefficient, dimensionless
P_{va} = pressure, in. of water
ρ = gas density in the measurement plane, lb$_m$/ft^3

7.8.2.3 Gas Volumetric Flow. The gas volumetric flow at the Pitot-static tube array measurement plane shall be obtained from Equation 7-19.

$$Q = V_a A \tag{7-19}$$

where

Q = gas volumetric flow rate, m^3/s [cfm]
V_a = average gas velocity, m/s [ft/min]
A = measurement plane cross section area, m^2 [ft^2]

7.8.3 Self-Averaging Probe Gas Flow Measurement. Self-averaging probes include multiple total and static pressure ports along a straight line or around a circumference within the gas stream. The self-averaged total pressure is connected to one side of a differential pressure transducer, and the self-averaged static pressure is connected to the other side of the same pressure transducer.

(Informative Note: For further reading, see Informative Appendix A Reference A8.)

7.8.3.1 Average Velocity Pressure. The average velocity pressure shall be obtained from Equation 7-20.

$$P_{va} = P_{ta} - P_{sa} \tag{7-20}$$

where

P_{va} = average velocity pressure, Pa (in. of water)
P_{ta} = measured average total pressure, Pa (in. of water)
P_{sa} = measured average static pressure, Pa in. (in. of water)

7.8.3.2 Average Gas Velocity. The average gas velocity shall be obtained from Equation 7-21 in SI units or from Equation 7-22 in I-P units:

In SI units:

$$V_a = K_4\sqrt{\frac{2P_{va}}{\rho}} \tag{7-21}$$

where

V_a = average gas velocity, m/s

K_4 = calibration coefficient, dimensionless
P_{va} = average velocity pressure, Pa
ρ = gas density, kg/m^3

In I-P units:

$$V_a = 1097.8\, K_4 \sqrt{\frac{P_{va}}{\rho}} \tag{7-22}$$

where

V_a = average gas velocity, ft/min
K_4 = calibration coefficient, dimensionless
P_{va} = average velocity pressure, in. of water
ρ = gas density in the measurement plane, lb$_m$/ft^3

7.8.3.3 Gas Volumetric Flow. The gas volumetric flow at the Pitot-static tube array measurement plane shall be obtained from Equation 7-23.

$$Q = V_a A \tag{7-23}$$

where

Q = gas volumetric flow rate, m^3/s [cfm]
V_a = average gas velocity, m/s [ft/min]
A = measurement plane cross section area, m^2 [ft^2]

7.9 Vortex-Shedding Flowmeters. Review Section 7.1. Vortex-shedding flowmeters are used to determine gas velocities. Piezoelectric methods, strain-gage methods, or hot-film methods are used to sense dynamic pressure variations created by vortex shedding. The operating principle for these flowmeters is based on vortex shedding that occurs downstream of an immersed blunt-shaped solid body. As the gas stream passes a blunt-shaped body, the gas separates and generates small vortices that are shed alternately along and downstream of each side of the blunt-shaped body. Each vortex-shedding meter is designed to have a known relationship between the Strouhal number and the Reynolds number so that the vortex shedding frequency is a known function of the gas flow velocity over a specified flow velocity range.

(Informative Note: For further reading, see Informative Appendix Reference A7.)

8. UNCERTAINTY REQUIREMENTS.

8.1 Post-Test Uncertainty Analysis. A post-test analysis of the measurement system uncertainty, performed in accordance with ANSI/ASME PTC 19.1[1], shall accompany each gas mass flow rate measurement if specified in the test plan in Section 5.1. Installation effects on the accuracy of the instrument shall be included in the uncertainty analysis for each installation that does not conform to the instrument manufacturer's installation requirements.

(Informative Note: Informative Appendix B contains an example of uncertainty calculations.)

8.2 Method to Express Uncertainty. Assumptions, parameters, and calculations used in estimating uncertainty shall be clearly documented prior to expressing uncertainty values. Uncertainty shall be expressed as shown in Equation 8-1.

$$v = \bar{X}_m \pm U_{\bar{X}}\ (P\%) \tag{8-1}$$

where:

v = the variable that is a measurement or a calculated result
$\overline{X}_m$ = the best estimate of the true value
$U_{\overline{X}}$ = the uncertainty estimate for the variable
P = the confidence level, ~~percent~~ dimensionless

(Informative Note: For example: gas mass flow rate = 2.538 kg/s ± 0.013 kg/s (335.7 lb_m/min ± 1.7 lb_m/min); 95% states that the measured gas flow is believed to be 2.538 kg/s (335.7 lb_m/min) with a 95% probability that the true value lies within ±0.013 kg/s (±1.7 lb_m/min) of this value.)

9. TEST REPORT

If the test plan in Section 5.1 defines the test report requirements, the test report requirements in the test plan supersedes all of the requirements in Section 9. Otherwise, Section 9 specifies the test report requirements.

9.1 Test Identification

a. Date, place, and time.
b. Operator.

9.2 Unit Under Test Description

a. Model number and serial number.
b. Gas specification.
c. Source of gas properties.

9.3 Instrument Description

a. Flowmeter description, model number, serial number, and location within the UUT.
b. Operating range.
c. Instrument accuracy based on specifications or calibration.
d. Documentational evidence of instrument calibrations.

9.4 Measurement System Description

a. Description of instrument installation specifics.
b. Measurement system accuracy based on specifications or calibration.
c. Documentational evidence of instrument calibrations.

9.5 Test Conditions

a. Test conditions in accordance with the test plan in Section 5.1
b. Ambient temperature, ºC (ºF).
c. Barometric pressure Pa, (psia) if pressures instruments are measuring gauge pressure.

9.6 Test Results If Required by the Test Plan in Section 5.1

9.6.1 Gas mass flow rate unless otherwise specified by the test plan:
a. Gas mass flow rate, kg/s (lb_m/min).
b. Pretest uncertainty in gas mass flow rate, kg/s (lb_m/min).
c. Post-test uncertainty in gas mass flow rate, kg/s (lb_m/min).

9.6.2 Gas volumetric flow rate if required by the test plan:
a. Gas volumetric flow rate, m^3/s (cfm).
b. Pretest uncertainty in gas volumetric flow rate, m^3/s (cfm).
c. Post-test uncertainty in gas volumetric flow rate, m^3/s (cfm).

10. REFERENCES

1. ANSI/ASHRAE Standard 41.2-2022, *Standard Methods for Air Velocity and Airflow Measurements,* Atlanta, GA.
2. ANSI/ASHRAE Standard 41.10-2024, *Standard Methods for Refrigerant Volumetric or Mass Flow Measurements Using Flowmeters,* Atlanta, GA.
3. ANSI/ASHRAE Standard 51-2016 (AMCA 210-16), *Laboratory Methods of Testing Fans for Certified Aerodynamic Performance Rating,* Atlanta, GA.
4. ASME PTC 19.1-2018, *Test Uncertainty*, ASME, New York, NY.
5. NIST *Standard Reference Database 23: NIST Reference Fluid Thermodynamic and Transport Properties Database (REFPROP) Version 10, National Institute of Standards and Technology, Gaithersburg, MD.*
6. ANSI/ASHRAE Standard 41.1-2024, *Standard Methods for Temperature Measurement*. ASHRAE, Atlanta, GA. See Note 1.
7. ANSI/ASHRAE Standard 41.3-2021*, Standard Methods for Pressure Measurement*. ASHRAE, Atlanta, GA. See Note 2
8. ANSI/ASME PTC 19.5-2022*, Flow Measurement.* ASME, New York, NY. (See Note 3.)
9. ANSI/ASME MFC-3M-2022 , *Measurement of Fluid Flow in Pipes Using Orifice, Nozzle, and Venturi*. ASME, New York, NY. See Note 3.

Note 1: Reference 6 is not required if there are no temperature measurements.
Note 2: Reference 7 is not required if there are no pressure measurements.
Note 3: References 8 and 9 are only required if using an Orifice, Flow Nozzle, or Venturi Tube.

(This appendix is not part of this standard. It is merely informative and does not contain requirements necessary for conformance to the standard. It has not been processed according to the ANSI requirements for a standard and may contain material that has not been subject to public review or a consensus process. Unresolved objectors on informative material are not offered the right to appeal at ASHRAE or ANSI.)

INFORMATIVE APPENDIX A: BIBLIOGRAPHY

A1 Kelley, Jeffrey D., and Hedengren, John D., "A Steady-State Detection (SSD) Algorithm to Detect Non-Stationary Drifts in Processes," BYU Scholars Archive, 2013.

A2 Miller, Steven J., "The Method of Least Squares," Brown University, 2006.

A3 ANSI/ASME MFC-11M-2006 (2014), *Measurement of Fluid Flow by Means of Coriolis Mass Flowmeters.* ASME, New York, NY.

A4 Doebelin, E. O., *Measurement Systems - Application and Design,* Fifth Edition. 2004. McGraw-Hill Book Company.

A5 ASME MFC-4M-1986 (R2016), *Measurement of Gas by Turbine Meters.* ASME, New York, NY.

A6 ANSI/ASME MFC-6M-2013, *Measurement of Fluid Flow in Pipes Using Vortex Flow Meters.* 1998. ASME, New York, NY

A7 ANSI/ASME MFC-18M-2001(R2013), *Measurement of Fluid Flow Using Variable Area Meters.* ASME, New York, NY. .

A8 ISO/TR 5168-2005, *Measurement of Fluid Flow - Evaluation on Uncertainties*. 1998. International Organization for Standardization, Geneva, Switzerland.

A9 ANSI/ASME MFC-12M-2006 (R2014), *Measurement of Fluid Flow in Closed Conduits Using Multiport Averaging Pitot Primary Elements*. ASME, New York, NY.

A10 ANSI/ASME Standard MFC-10M-2000 (R2011), *Method for Establishing Installation Effects on Flowmeters.* ASME, New York, NY.

A11 ANSI/ASME MFC-2M-1983 (R2013), *Measurement Uncertainty for Fluid Flow in Closed Circuits*. ASME, New York, NY.

(This appendix is not part of this standard. It is merely informative and does not contain requirements necessary for conformance to the standard. It has not been processed according to the ANSI requirements for a standard and may contain material that has not been subject to public review or a consensus process. Unresolved objectors on informative material are not offered the right to appeal at ASHRAE or ANSI.)

INFORMATIVE APPENDIX B: AN UNCERTAINTY ANALYSIS EXAMPLE FOR A DIFFERENTIAL PRESSURE FLOWMETER

Following ASME PTC 19.1-2018 - *Test Uncertainty* procedures and terminology, estimate the uncertainty interval, at 95% confidence, about the measured result of a long-radius flow nozzle. Because long-radius flow nozzles require measurements followed by calculations to produce mass flow rate results, Section 5, *Uncertainty of a Measurement,* will be followed to determine individual measurement uncertainty that is fed into Section 6, *Uncertainty of a Result Calculated from Multiple Parameters*, for use in determining uncertainty propagation producing a final uncertainty estimate for the mass flow rate. An ASME long-radius stainless-steel flow nozzle, in accordance with ANSI/ASME PTC 19.5, is installed in a laboratory test facility that will be used to measure superheated steam flow at a nominal pressure of 379.2 kPa (55 psi) and temperature of 160 °C (320 °F). Reported uncertainty units are kg/s (lb_m/s).

7.4.2.5 Nozzle Inlet Gas Properties. The nozzle inlet gas density ρ_1, constant pressure specific heat c_p and constant volume specific heat c_p shall be obtained from the gas property data prescribed in Section 5.3.8 as a function of the nozzle inlet gas temperature t_1 and pressure p_1 at each data point.

7.4.2.6 Nozzle Gas Volumetric Flow Rate. The nozzle gas volumetric flow rate shall be calculated from Equation 7-6 in SI units or Equation 7-7 in I-P units.

$$Q = C\varepsilon AK_1\sqrt{\frac{2(\Delta p)}{\rho_1(1-E\beta^4)}} \tag{7-6}$$

Where

Q = nozzle gas volumetric flow rate, m^3/s
C = nozzle discharge coefficient, dimensionless
ε = nozzle expansibility factor, dimensionless
A = nozzle throat area, m^2
K_1 = nozzle calibration coefficient = 1.038, dimensionless
ρ_1 = nozzle inlet gas density, kg/m^3
ΔP = nozzle differential pressure, Pa
E = flow kinetic energy coefficient = 1.043, dimensionless
β = nozzle ratio, dimensionless

$$Q = 0.47268\ \times C\varepsilon AK_1\sqrt{\frac{2(\Delta p)}{\rho_1(1-E\beta^4)}} \tag{7-7}$$

Where

Q = nozzle gas volumetric flow rate, ft^3/s
C = nozzle discharge coefficient, dimensionless
ε = nozzle expansibility factor, dimensionless
A = nozzle throat area, in^2
K_1 = nozzle calibration coefficient = 1.038, dimensionless

ρ_1 = nozzle inlet gas density, lb_m/ft^3
ΔP = nozzle differential pressure, psid
E = flow kinetic energy coefficient = 1.043, dimensionless
β = nozzle ratio, dimensionless
0.47268= units conversion coefficient, $\sqrt{\frac{(lb_m - ft^3)}{(psid - in^4 - s^2)}}$

7.4.2.7 Nozzle Expansibility Factor. The nozzle expansibility factor ε for a long-radius nozzle shall be obtained from Equation 7-8. This equation assumes that the gas is an ideal gas, and the gas flow is an isentropic process.

$$\varepsilon = \left[r^{\frac{2}{\gamma}} \left(\frac{\gamma}{\gamma - 1} \right) \left(\frac{1 - r^{\frac{\gamma - 1}{\gamma}}}{1 - r} \right) \left(\frac{1 - \beta^4}{1 - \beta^4 r^{\frac{2}{\gamma}}} \right) \right]^{1/2} \tag{7-8}$$

Where

$r = p_2/p_1$ absolute pressure ratio, dimensionless
$\gamma = c_p/c_v$ specific heat ratio, dimensionless

7.4.2.8 Nozzle Gas Mass Flow Rate. The nozzle gas mass flow rate shall be obtained from Equation 7-10 in kg/s (lb_m/s).

$$\dot{m} = \rho_1 Q \tag{7-10}$$

Table B-1. Independent Parameters Descriptions

Symbol	Independent Parameter	Units	Nominal Value	Expected Range	Initial Selected Instrument
d	Nozzle Throat Diameter	mm (inches)	20.32 (0.8)	10 to 30 (0.39 to 1.18)	Inside Diameter Micrometer
D	Pipe Diameter	mm (inches)	49.276 (1.94)	48.77 to 49.53 (1.92 to 2.00)	Inside Diameter Micrometer
t_1	Inlet Gas Temperature	ºC (ºF)	160 (320)	140 to 200 (284 to 392)	Type T thermocouple
p_1	Inlet Gas Pressure	kPa (psia)	379.2 (55)	275 to 825 (40 to 120)	Strain Gage Pressure Transducer
ΔP	Nozzle Pressure Drop	kPa (psid)	68.95 (10)	34.5 to 172.4 (5 to 25)	Strain Gage Differential Pressure Transducer
C	Discharge Coefficient	none	0.995	0 to 1	Calibration from an ISO 17025 Certified Lab

Table B-2. Relationships of the Calculated Parameters

Symbol	Calculated Parameter	Functional Relationship	Functional Calculation	Nominal Value	Units
A	Nozzle Throat Area Nozzle Ratio	$f(d)$	$\pi d^2 / 4$	0.0003243 (0.502655)	m^2 (in^2)
β	Absolute Throat Pressure Nozzle Throat Area	$f(d,D)$	d / D	0.4124	none
p_2	Nozzle Ratio	$f(p_1,\Delta P)$	$p_1 - \Delta P$	310.25 (45)	kPa (psia)

c_p	Constant Pressure Specific Heat	$f(p_1,t_1)$	Property Data	2.1766 (0.5199)	kJ/(kg-°C) (BTU/(lb$_m$-°F))
c_v	Constant Volume Specific Heat	$f(p_1,t_1)$	Property Data	1.6088 (0.3843)	kJ/(kg-°C) (BTU/(lb$_m$-°F))
ρ_1	Inlet Gas Density	$f(p_1,t_1)$	Property Data	1.9559 (0.1221)	kg/m^3 (lb$_m$/ft^3)
ε	Nozzle Expansibility Factor	$f(p_1,t_{1,},\Delta P,d,D)$	Equation 7-8	0.8906	none

Table B-3. Uncertainty of Each Measured Parameter

Parameter	Nominal Value	Error Origin	Calibration Error		Data Acquisition Error		Data Reduction Error	
			Elemental Systematic Standard Uncertainty	Elemental Random Standard Uncertainty	Elemental Systematic Standard Uncertainty	Elemental Random Standard Uncertainty	Elemental Systematic Standard Uncertainty	Elemental Random Standard Uncertainty
d, mm (in)	20.32 (0.80)	See Note 1 below	0.0305 (0.0012)	0	0	0	0	0
D, mm (in)	49.276 (1.94)	See Note 1 below	0.0508 (0.002)	0	0	0	0	0
t_1, °C (°F)	160 (320)	Calibration	0.194 (0.35)	0.028 (0.05)	0	0	0	0
		Ice Reference Junction	0	0	0.133 (0.24)	0.056 (0.1)	0	0
		Data Acquisition	0	0	0.167 (0.30)	0.044 (0.08)	0	0
		Data Reduction/ Curve Fit	0	0	0	0	0.061 (0.11)	0.009 (0.017)
p_1, kPa (psia)	379.2 (55)	Excitation Voltage	0.0010 (0.00015)	0.0007 (0.0001)	0	0	0	0
		Signal Conditioning	0.0172 (0.0025)	0.0062 (0.0009)	0	0	0	0
		Calibration	4.1369 (0.6)	0.0041 (0.0006)	0	0	0	0
		Data Acquisition	0	0	0.0276 (0.004)	0.0345 (0.005)	0	0

		Data Reduction/ Curve Fit	0	0	0	0	0.0083 (0.0012)	0
ΔP, kPa (psid)	68.95 (10)	Excitation Voltage	0.0103 (0.0015)	0.0014 (0.0002)	0	0	0	0
		Signal Conditioning	0.0758 (0.011)	0.0048 (0.0007)	0	0	0	0
		Calibration	0.4137 (0.060)	0.0138 (0.002)	0	0	0	0
		Data Acquisition	0	0	0.0434 (0.0063)	0.1655 (0.024)	0	0
		Data Reduction/ Curve Fit	0	0	0	0	0.1034 (0.015)	0
C	0.995	Uncertainty from an ISO17025 lab calibration	0.0025	0	0	0	0	0

Note 1 – Because this meter was calibrated, error effects in the throat and pipe diameters are eliminated provided the same dimensions used during the calibration are also used in the test calculations and they have not changed due to damage such as erosion. Calibration systematic uncertainties are included to allow the calculation of the systematic standard uncertainties.

Note 2 – Nominal values in Table B-3 are the results of ten (10) readings for each parameter. 30 readings would be required to be statistically significant, but users are likely to consider 30 readings to be a burden. 10 readings appear to be a reasonable compromise.

Estimation of the uncertainty interval, at 95% confidence, about the measured mass flow rate of a differential pressure flowmeter is done following a five-step procedure based on ASME PTC 19.1-2018 - *Test Uncertainty*. Each step is detailed using SI units and I-P units.

B1. Calculate the Random and Systematic Standard Uncertainties for Each Parameter

For each parameter in Equation 7-9, the random and systematic standard uncertainties must be first calculated. Following ASME PTC 19.1-2018 - *Test Uncertainty*, these uncertainties are respectively given as follows:

$$s_{\bar{X}} = \pm \frac{1}{\sqrt{N}} \sqrt{\sum_{k=1}^{K} (s_{\bar{X}_k})^2} \tag{B-1}$$

$$b_{\bar{X}} = \pm \sqrt{\sum_{k=1}^{K} (b_{\bar{X}_k})^2} \tag{B-2}$$

where $\bar{X}$ represents a parameter, K is the total number of elemental uncertainty sources for a given parameter, N is the number of readings, $s_{\bar{X}_k}$ is an elemental random standard uncertainty and $b_{\bar{X}_k}$ is an elemental systematic standard uncertainty. Numerical values of $s_{\bar{X}}$ and $b_{\bar{X}}$ are calculated in the next two sections using elemental standard uncertainties from Table B-3. Calculated values are presented with three significant figures after the decimal point; the exact values are used in calculations. The random and systematic standard uncertainties for t_1 and p_1 are calculated for later estimation of the uncertainty of the inlet gas density, constant pressure specific heat and constant volume specific heat.

B1.1 SI Units

Using Equations B-1 and B-2, and Table B-3, the following random and systematic standard uncertainties are calculated for each parameter:

$s_d = 0.000$ m $\quad b_d = 3.05 \times 10^{-5}$ m
$s_D = 0.000$ m $\quad b_D = 5.08 \times 10^{-5}$ m
$s_{t_1} = 0.0244$ °C $\quad b_{t_1} = 0.295$ °C
$s_{p_1} = 11.162$ Pa $\quad b_{p_1} = 4{,}137.036$ Pa
$s_{\Delta P} = 52.541$ Pa $\quad b_{\Delta P} = 435.402$ Pa
$s_C = 0.000$ $\quad b_C = 0.00250$

B1.2 I-P Units

Using Equations B-1 and B-2, and Table B-3, the following random and systematic standard uncertainties are calculated for each parameter:

$s_d = 0.000$ in $\quad b_d = 0.00120$ in
$s_D = 0.000$ in $\quad b_D = 0.00200$ in
$s_{t_1} = 0.0439$ °F $\quad b_{t_1} = 0.531$ °F
$s_{p_1} = 0.00162$ psia $\quad b_{p_1} = 0.600$ psia
$s_{\Delta P} = 0.00762$ psid $\quad b_{\Delta P} = 0.0631$ psid
$s_C = 0.000$ $\quad b_C = 0.00250$

B2. Calculate the Random and Systematic Standard Uncertainties of the Working Fluid Properties

The random and systematic standard uncertainties for ρ_1, c_p and c_v cannot be calculated directly from Table B-3 unlike the other parameters in Equation 7-9. However, the random and systematic standard uncertainties were calculated for state variables t_1 and p_1 and will be used here to calculate the random and systematic standard uncertainties of ρ_1, c_p and c_v. Extremum values of t_1 and p_1, within a 95% confidence interval, must be calculated to estimate extremum values of ρ_1, c_p and c_v, which are then used to estimate the random and systematic standard uncertainties of ρ_1, c_p and c_v. In order to determine the 95% confidence interval of t_1 and p_1, the combined and expanded uncertainties are calculated. These uncertainties are respectively given as follows:

$$u_{\bar{X}} = \pm\sqrt{(s_{\bar{X}})^2 + (b_{\bar{X}})^2} \tag{B-3}$$

$$U_{\bar{X}} = k \times u_{\bar{X}} \tag{B-4}$$

where k is the coverage factor. The coverage factor value is selected based on the appropriate degrees of freedom and confidence level in the Student's t table – see ASME PTC 19.1-2018 - *Test Uncertainty* Annex B. The degree of freedom used is a combined degree of freedom on the separate degree of freedom for the random standard uncertainties and elemental systematic standard uncertainties. If the degrees of freedom for the random standard uncertainties and the elemental systematic standard uncertainties are known or estimated, one can refer to ASME PTC 19.1-2018 - *Test Uncertainty* Annex B to calculate the exact degree of freedom. In most engineering applications, a coverage factor of 2 is used for 95% confidence intervals, assuming a large degrees of freedom.

B2.1 SI Units

Assuming a large degree of freedom and using the previously calculated random and systematic standard uncertainties of t_1 and p_1, the expanded uncertainties are:

$U_{t_1} = 0.592$ °C
$U_{p_1} = 8{,}274.103$ Pa

The extreme values of t_1 and p_1 and the corresponding values of ρ_1, c_p and c_v are presented in Table B-4.1.

Table B-4.1. Gas properties extreme values – SI units

	Minimum value	**Maximum value**
t_1 (°C)	159.704	160.296
p_1 (kPa)	375.063	383.337
ρ_1 (kg/m^3)	1.935	1.976
c_p (kJ/(kg°C))	2.174	2.179
c_v (kJ/(kg°C))	1.607	1.610

Based on previous experience, the variations for ρ_1, c_p and c_v are assumed to be entirely systematic errors. Using Equation B-3 and defining half the difference between ρ_1, c_p and c_v extremums as equal to the expanded uncertainty (Equation B-4), the random and systematic standard uncertainties are:

s_{ρ_1} = 0.000 kg/m^3
b_{ρ_1} = 0.0102 kg/m^3
s_{c_p} = 0.000 kJ/(kg°C)
b_{c_p} = 0.00123 kJ/(kg°C)
s_{c_v} = 0.000 kJ/(kg°C)
b_{c_v} = 0.000725 kJ/(kg°C)

B2.2 I-P Units

Assuming a large degree of freedom and using the previously calculated random and systematic standard uncertainties of t_1 and p_1, the expanded uncertainties are:

U_{t_1} = 1.065 °F
U_{p_1} = 1.200 psia

The extreme values of t_1 and p_1 and the corresponding values of ρ_1, c_p and c_v are presented in Table B-4.2.

Table B-4.2. Gas properties extreme values – I-P units

	Minimum value	**Maximum value**
t_1 (°F)	319.468	320.533
p_1 (psia)	54.400	55.600
ρ_1 (lb$_m$/ft^3)	0.121	0.123
c_p (BTU/(lb$_m$°F))	0.519	0.520
c_v (BTU/(lb$_m$°F))	0.384	0.385

Based on previous experience, the variations for ρ_1, c_p and c_v are assumed to be entirely systematic errors. Using Equation B-3 and defining half the difference between ρ_1, c_p and c_v extremums as equal to the expanded uncertainty (Equation B-4), the random and systematic standard uncertainties are:

s_{ρ_1} = 0.000 lb$_m$/ft^3
b_{ρ_1} = 0.000640 lb$_m$/ft^3
s_{c_p} = 0.000 BTU/(lb$_m$°F)
b_{c_p} = 0.000293 BTU/(lb$_m$°F)
s_{c_v} = 0.000 BTU/(lb$_m$°F)
b_{c_v} = 0.000173 BTU/(lb$_m$°F)

B3. Calculate and Evaluate the Partial Derivative of the Mass Flow Rate for Each Parameter

The random and systematic standard uncertainties of the mass flow rate are a non-linear combination of the random and systematic standard uncertainties of the parameters in Equation 7-9. The random and systematic standard uncertainties of the mass flow rate are respectively given as follows:

$$s_{\dot{m}} = \pm\sqrt{\sum_{i=1}^{l}\left(\theta_i s_{\bar{X}_i}\right)^2 + (random\ correlation\ terms)} \qquad \text{(B-5)}$$

$$b_{\dot{m}} = \pm\sqrt{\sum_{i=1}^{l}\left(\theta_i b_{\bar{X}_i}\right)^2 + (systematic\ correlation\ terms)} \qquad \text{(B-6)}$$

where θ_i is the evaluated partial derivative of $\dot{m}$ with respect to the ith parameter, l is the number of parameters in Equation 7-9, $s_{\bar{X}_i}$ and $b_{\bar{X}_i}$ are the corresponding random and systematic standard uncertainties of the ith parameter previously calculated in Sections B1.1 and B1.2. If necessary, correlation terms can be added for both the random and systematic standard uncertainty of the mass flow rate. These terms account for random and systematic standard uncertainties that are not independent of one another for a given parameter. If these terms are needed, one can refer to Section 7 of ASME PTC 19.1-2018 - *Test Uncertainty* for the methodology. Explicitly, θ_i is expressed as follows:

$$\theta_i = \frac{\partial \dot{m}}{\partial \bar{X}_i} \qquad \text{(B-7)}$$

Once calculated, the partial derivative is then evaluated considering the value of each parameter. In Equation 7-9, A must be expressed in terms of d, β must be expressed in terms of d and D, and ε must be expressed in terms of d, D, ΔP, p_1, c_p and c_v. Partial derivatives are calculated and evaluated in the next two sections. Note that, in general, using a commercial equation solver software, such as MATLAB or EES, significantly reduces the time and effort required to complete an uncertainty analysis.

B3.1 Derivations of the Partial Derivatives in SI Units

Due to the derivation complexity of most of the partial derivatives of Equation 7-9, this section does not provide a step-by-step procedure with partial results, but only the final partial derivatives are given using a commercial equation solver software.

B3.1.1 Derive the Partial Derivative $\frac{\partial \dot{m}}{\partial d}$

$$\frac{\partial \dot{m}}{\partial d} = \mathrm{A} + \mathrm{B} + \mathrm{C} \qquad \text{(B-8)}$$

Where

$$\mathrm{A} = \frac{2^{\frac{1}{2}} * C * K_1 * d * \Delta P^{\frac{1}{2}} * \rho_1^{\frac{1}{2}} * \pi * \left(-\frac{1}{\frac{E*d^4}{D^4} - 1}\right)^{\frac{1}{2}} * \left(-\frac{C_p*\left(\frac{d^4}{D^4} - 1\right)*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v} - 1\right)}{C_p}} - 1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*\left(\frac{\Delta P - p_1}{p_1} + 1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4} - 1\right)*\left(\frac{C_p}{C_v} - 1\right)}\right)^{\frac{1}{2}}}{2} \qquad \text{(B-9)}$$

$$B = -\frac{2^{\frac{1}{2}}*C*K_1*d^2*\Delta P^{\frac{1}{2}}*\rho_1^{\frac{1}{2}}*\pi*\left(\frac{4*C_p*d^3*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*D^4*\left(\frac{\Delta P - p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)}-\frac{4*C_p*d^3*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{4*C_v}{C_p}}}{C_v*D^4*\left(\frac{\Delta P - p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)^2*\left(\frac{C_p}{C_v}-1\right)}\right)*\left(-\frac{1}{\frac{E*d^4}{D^4}-1}\right)^{\frac{1}{2}}}{8*\left(-\frac{C_p*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*\left(\frac{\Delta P - p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)}\right)^{\frac{1}{2}}} \qquad \text{(B-10)}$$

$$C = \frac{2^{\frac{1}{2}}*C*E*K_1*d^5*\Delta P^{\frac{1}{2}}*\rho_1^{\frac{1}{2}}*\pi*\left(-\frac{C_p*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*\left(\frac{\Delta P - p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)}\right)^{\frac{1}{2}}}{2*D^4*\left(\frac{E*d^4}{D^4}-1\right)^2*\left(-\frac{1}{\frac{E*d^4}{D^4}-1}\right)^{\frac{1}{2}}} \qquad \text{(B-11)}$$

B3.1.2 Derive the Partial Derivative $\frac{\partial \dot{m}}{\partial D}$

$$\frac{\partial \dot{m}}{\partial D} = \mathrm{A} + \mathrm{B} \qquad \text{(B-12)}$$

Where

$$A = \frac{2^{\frac{1}{2}}*C*K_1*d^2*\Delta P^{\frac{1}{2}}*\rho_1^{\frac{1}{2}}*\pi*\left(\frac{4*C_p*d^4*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*D^5*\left(\frac{\Delta P - p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)}-\frac{4*C_p*d^4*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{4*C_v}{C_p}}}{C_v*D^5*\left(\frac{\Delta P - p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)^2*\left(\frac{C_p}{C_v}-1\right)}\right)*\left(-\frac{1}{\frac{E*d^4}{D^4}-1}\right)^{\frac{1}{2}}}{8*\left(-\frac{C_p*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*\left(\frac{\Delta P - p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)}\right)^{\frac{1}{2}}}$$

(B-13)

$$B = -\frac{2^{\frac{1}{2}}*C*E*K_1*d^6*\Delta P^{\frac{1}{2}}*\rho_1^{\frac{1}{2}}*\pi*\left(-\frac{C_p*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*\left(\frac{\Delta P - p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)}\right)^{\frac{1}{2}}}{2*D^5*\left(\frac{E*d^4}{D^4}-1\right)^2*\left(-\frac{1}{\frac{E*d^4}{D^4}-1}\right)^{\frac{1}{2}}} \qquad \text{(B-14)}$$

B3.1.3 Derive the Partial Derivative $\frac{\partial \dot{m}}{\partial C}$

$$\frac{\partial\dot{m}}{\partial C}=\frac{2^{\frac{1}{2}}*K_1*d^2*\Delta P^{\frac{1}{2}}*\rho_1^{\frac{1}{2}}*\pi*\left(-\frac{1}{\frac{E*d^4}{D^4}-1}\right)^{\frac{1}{2}}*\left(-\frac{Cp*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{Cv*\left(\frac{Cp}{Cv}-1\right)}{Cp}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*Cv}{Cp}}}{Cv*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*Cv}{Cp}}}{D^4}-1\right)*\left(\frac{Cp}{Cv}-1\right)}\right)^{\frac{1}{2}}}{4} \tag{B-15}$$

B3.1.4 Derive the Partial Derivative $\frac{\partial\dot{m}}{\partial\rho_1}$

$$\frac{\partial\dot{m}}{\partial\rho_1}=\frac{2^{\frac{1}{2}}*C*K_1*d^2*\Delta P^{\frac{1}{2}}*\pi*\left(-\frac{1}{\frac{E*d^4}{D^4}-1}\right)^{\frac{1}{2}}*\left(-\frac{Cp*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{Cv*\left(\frac{Cp}{Cv}-1\right)}{Cp}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*Cv}{Cp}}}{Cv*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*Cv}{Cp}}}{D^4}-1\right)*\left(\frac{Cp}{Cv}-1\right)}\right)^{\frac{1}{2}}}{8*\rho_1^{\frac{1}{2}}} \tag{B-16}$$

B3.1.5 Derive the Partial Derivative $\frac{\partial\dot{m}}{\partial\Delta P}$

$$\frac{\partial\dot{m}}{\partial\Delta P}=\mathrm{A}+\frac{\mathrm{B}\ (\mathrm{C+D+E+F})}{\mathrm{G}} \tag{B-17}$$

Where

$$\mathrm{A}=\frac{2^{\frac{1}{2}}*C*K_1*d^2*\rho_1^{\frac{1}{2}}*\pi*\left(-\frac{1}{\frac{E*d^4}{D^4}-1}\right)^{\frac{1}{2}}*\left(-\frac{Cp*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{Cv*\left(\frac{Cp}{Cv}-1\right)}{Cp}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*Cv}{Cp}}}{Cv*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*Cv}{Cp}}}{D^4}-1\right)*\left(\frac{Cp}{Cv}-1\right)}\right)^{\frac{1}{2}}}{8*\Delta P^{\frac{1}{2}}} \tag{B-18}$$

$$\mathrm{B}=2^{\frac{1}{2}}*C*K_1*d^2*\Delta P^{\frac{1}{2}}*\rho_1^{\frac{1}{2}}*\pi*\left(-\frac{1}{\frac{E*d^4}{D^4}-1}\right)^{\frac{1}{2}} \tag{B-19}$$

$$\mathrm{C}=\frac{\left(\frac{d^4}{D^4}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*Cv}{Cp}}*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{Cv*\left(\frac{Cp}{Cv}-1\right)}{Cp}-1}}{p_1*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*Cv}{Cp}}}{D^4}-1\right)} \tag{B-20}$$

$$D = \frac{2*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}-1}}{p_1*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)} \tag{B-21}$$

$$E = \frac{C_p*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*p_1*\left(\frac{\Delta P-p_1}{p_1}+1\right)^2*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)} \tag{B-22}$$

$$F = -\frac{2*d^4*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}-1}}{D^4*p_1*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)^2*\left(\frac{C_p}{C_v}-1\right)} \tag{B-23}$$

$$G = 8*\left(-\frac{C_p*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)}\right)^{\frac{1}{2}} \tag{B-24}$$

B3.1.6 Derive the Partial Derivative $\frac{\partial \dot{m}}{\partial p_1}$

$$\frac{\partial \dot{m}}{\partial p_1} = \frac{\mathrm{A\ (B+C+D+E)}}{\mathrm{F}} \tag{B-25}$$

Where

$$A = -2^{\frac{1}{2}} * C * K_1 * d^2 * \Delta P^{\frac{1}{2}} * \rho_1^{\frac{1}{2}} * \pi * \left(-\frac{1}{\frac{E*d^4}{D^4}-1}\right)^{\frac{1}{2}} \tag{B-26}$$

$$B = \frac{\left(\frac{d^4}{D^4}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}-1}*\left(\frac{\Delta P-p_1}{p_1^2}+\frac{1}{p_1}\right)}{\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)} \tag{B-27}$$

$$C = \frac{2*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}-1}*\left(\frac{\Delta P - p_1}{p_1^2}+\frac{1}{p_1}\right)}{\left(\frac{\Delta P - p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)} \tag{B-28}$$

$$D = \frac{C_p*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}*\left(\frac{\Delta P - p_1}{p_1^2}+\frac{1}{p_1}\right)}{C_v*\left(\frac{\Delta P - p_1}{p_1}+1\right)^2*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)} \tag{B-29}$$

$$E = -\frac{2*d^4*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}-1}*\left(\frac{\Delta P - p_1}{p_1^2}+\frac{1}{p_1}\right)}{D^4*\left(\frac{\Delta P - p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)^2*\left(\frac{C_p}{C_v}-1\right)} \tag{B-30}$$

$$F = 8*\left(-\frac{C_p*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*\left(\frac{\Delta P - p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)}\right)^{\frac{1}{2}} \tag{B-31}$$

B3.1.7 Derive the Partial Derivative $\frac{\partial \dot{m}}{\partial c_p}$

$$\frac{\partial \dot{m}}{\partial c_p} = \frac{\mathrm{A\ (B{+}C{+}D{+}E{+}F)}}{\mathrm{G}} \tag{B-32}$$

Where

$$A = -2^{\frac{1}{2}} * C * K_1 * d^2 * \Delta P^{\frac{1}{2}} * \rho_1^{\frac{1}{2}} * \pi * \left(-\frac{1}{\frac{E*d^4}{D^4}-1}\right)^{\frac{1}{2}} \tag{B-33}$$

$$B = \frac{\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*\left(\frac{\Delta P - p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P - p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)} \tag{B-34}$$

$$C = -\frac{2*\log\left(-\frac{\Delta P-p_1}{p_1}\right)*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_p*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)} \tag{B-35}$$

$$D = -\frac{C_p*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v^2*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)^2} \tag{B-36}$$

$$E = \frac{2*d^4*\log\left(-\frac{\Delta P-p_1}{p_1}\right)*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{4*C_v}{C_p}}}{C_p*D^4*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)^2*\left(\frac{C_p}{C_v}-1\right)} \tag{B-37}$$

$$F = \frac{C_p*\log\left(-\frac{\Delta P-p_1}{p_1}\right)*\left(\frac{d^4}{D^4}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}*\left(\frac{1}{C_p}-\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p^2}\right)}{C_v*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)} \tag{B-38}$$

$$G = 8*\left(-\frac{C_p*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)}\right)^{\frac{1}{2}} \tag{B-39}$$

B3.1.8 Derive the Partial Derivative $\frac{\partial \dot{m}}{\partial c_v}$

$$\frac{\partial \dot{m}}{\partial c_v} = \frac{\mathrm{A\ (B+C+D+E+F)}}{\mathrm{G}} \tag{B-40}$$

Where

$$A = 2^{\frac{1}{2}} * C * K_1 * d^2 * \Delta P^{\frac{1}{2}} * \rho_1^{\frac{1}{2}} * \pi * \left(-\frac{1}{\frac{E*d^4}{D^4}-1}\right)^{\frac{1}{2}} \tag{B-41}$$

$$\mathrm{B} = \frac{C_p*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v^2*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)} \tag{B-42}$$

$$\mathrm{C} = -\frac{2*\log\left(-\frac{\Delta P-p_1}{p_1}\right)*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)} \tag{B-43}$$

$$\mathrm{D} = -\frac{C_p^2*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v^3*\left(\frac{-\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)^2} \tag{B-44}$$

$$\mathrm{E} = \frac{2*d^4*\log\left(-\frac{\Delta P-p_1}{p_1}\right)*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{4*C_v}{C_p}}}{C_v*D^4*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)^2*\left(\frac{C_p}{C_v}-1\right)} \tag{B-45}$$

$$\mathrm{F} = \frac{C_p*\log\left(-\frac{\Delta P-p_1}{p_1}\right)*\left(\frac{d^4}{D^4}-1\right)*\left(\frac{1}{C_v}-\frac{\frac{C_p}{C_v}-1}{C_p}\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)} \tag{B-46}$$

$$\mathrm{G} = 8*\left(-\frac{C_p*\left(\frac{d^4}{D^4}-1\right)*\left(\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{C_v*\left(\frac{C_p}{C_v}-1\right)}{C_p}}-1\right)*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{C_v*\left(\frac{\Delta P-p_1}{p_1}+1\right)*\left(\frac{d^4*\left(-\frac{\Delta P-p_1}{p_1}\right)^{\frac{2*C_v}{C_p}}}{D^4}-1\right)*\left(\frac{C_p}{C_v}-1\right)}\right)^{\frac{1}{2}} \tag{B-47}$$

B3.2 Evaluation of Partial Derivatives in SI Units

$$\frac{\partial \dot{m}}{\partial d} = 15.844 \text{ kg/(m-s)} \tag{B-48}$$
$$\frac{\partial \dot{m}}{\partial D} = -0.149 \text{ kg/(m-s)} \tag{B-49}$$
$$\frac{\partial \dot{m}}{\partial C} = 0.158 \text{ kg/s} \tag{B-50}$$
$$\frac{\partial \dot{m}}{\partial \rho_1} = 0.0402 \text{ m}^3\text{/s} \tag{B-51}$$
$$\frac{\partial \dot{m}}{\partial \Delta P} = 8.47 \times 10^{-7} \text{ kg/(Pa-s)} \tag{B-52}$$
$$\frac{\partial \dot{m}}{\partial p_1} = 5.33 \times 10^{-8} \text{ kg/(Pa-s)} \tag{B-53}$$
$$\frac{\partial \dot{m}}{\partial c_p} = 0.00830 \text{ (kg}^2\text{-°C)/(kJ-s)} \tag{B-54}$$
$$\frac{\partial \dot{m}}{\partial c_v} = -0.0112 \text{ (kg}^2\text{-°C)/(kJ-s)} \tag{B-55}$$

B3.3 Evaluation of Partial Derivatives in I-P Units

The calculated partial derivatives in I-P units are the same as in SI units; however, a factor of 0.47268 must multiply each partial derivative for the correct unit conversion.

$$\frac{\partial \dot{m}}{\partial d} = 0.887 \text{ lb}_\text{m}\text{/(in-s)} \tag{B-56}$$
$$\frac{\partial \dot{m}}{\partial D} = -0.00833 \text{ lb}_\text{m}\text{/(in-s)} \tag{B-57}$$
$$\frac{\partial \dot{m}}{\partial C} = 0.349 \text{ lb}_\text{m}\text{/s} \tag{B-58}$$
$$\frac{\partial \dot{m}}{\partial \rho_1} = 1.420 \text{ ft}^3\text{/s} \tag{B-59}$$
$$\frac{\partial \dot{m}}{\partial \Delta P} = 0.0129 \text{ lb}_\text{m}\text{/(psid-s)} \tag{B-60}$$
$$\frac{\partial \dot{m}}{\partial p_1} = 8.11 \times 10^{-4} \text{ lb}_\text{m}\text{/(psia-s)} \tag{B-61}$$
$$\frac{\partial \dot{m}}{\partial c_p} = 0.0766 \text{ (lb}_\text{m}^2\text{-°F)/(BTU-s)} \tag{B-62}$$
$$\frac{\partial \dot{m}}{\partial c_v} = -0.104 \text{ (lb}_\text{m}^2\text{-°F)/(BTU-s)} \tag{B-63}$$

B4. Calculate the Random and Systematic Standard Uncertainties of the Mass Flow Rate

Equations B-5 and B-6 are evaluated in the Sections B4.1 and B4.2.

B4.1 SI Units

$$s_{\dot{m}} = 4.45 \times 10^{-5} \text{ kg/s} \tag{B-64}$$
$$b_{\dot{m}} = 8.63 \times 10^{-4} \text{ kg/s} \tag{B-65}$$

B4.2 I-P Units

$$s_{\dot{m}} = 9.82 \times 10^{-5} \text{ lb}_\text{m}\text{/s} \tag{B-66}$$
$$b_{\dot{m}} = 0.00190 \text{ lb}_\text{m}\text{/s} \tag{B-67}$$

B5. Calculate the Combined and Expanded Uncertainties of the Mass Flow Rate

Equations B-3 and B-4 are evaluated for the mass flow rate in Sections B5.1 and B5.2. A large degree of freedom is considered (k = 2).

B5.1 SI Units

$$u_{\dot{m}} = 8.64 \times 10^{-4} \text{ kg/s} \tag{B-68}$$

$U_{\dot{m}} = 0.00173$ kg/s (B-69)

The mass flow rate, at 95% confidence level, is:
$\dot{m} =$ (0.15731 ± 0.00173) kg/s (B-70)

B5.2 I-P Units
$u_{\dot{m}} = 0.00191$ lb_m/s (B-71)
$U_{\dot{m}} = 0.00381$ lb_m/s (B-72)

The mass flow rate, at 95% confidence level, is:
$\dot{m} =$ (0.34680 ± 0.00381) lb_m/s (B-73)

(This appendix is not part of this standard. It is merely informative and does not contain requirements necessary for conformance to the standard. It has not been processed according to the ANSI requirements for a standard and may contain material that has not been subject to public review or a consensus process. Unresolved objectors on informative material are not offered the right to appeal at ASHRAE or ANSI.)

INFORMATIVE APPENDIX C: FLOWMETER ACCURACY COMPARISONS

Table C-1 provides examples of the accuracy of commercial refrigerant flowmeters for comparison purposes.

TABLE C-1 Examples of commercial gas flowmeter accuracies for comparison purposes

Gas Flow Measurement Method	Gas Flow Measurement Accuracy
Coriolis Flowmeters	±0.1%
Thermal Flowmeters	±1%
Volume-Displacement Flowmeters	±0.5%
Orifice Meters	±0.5%
Flow Nozzles	±0.5%
Venturi Tubes	±0.5%
Turbine Flowmeters	±0.25%
Variable-Area Flowmeters	±2%
Ultrasonic Flowmeters	±1%
Vortex-Shedding Flowmeters	±0.75%
Drag-Force Flowmeters	±2%