BSR/ASHRAE Standard 41.7-2015R

Public Review Draft

Standard Methods for Gas Flow Measurement

First Public Review (November 2020)
(Complete Draft for Full Review)

This draft has been recommended for public review by the responsible project committee. To submit a comment on this proposed standard, go to the ASHRAE website at www.ashrae.org/standards-research--technology/public-review-drafts and access the online comment database. The draft is subject to modification until it is approved for publication by the Board of Directors and ANSI. Until this time, the current edition of the standard (as modified by any published addenda on the ASHRAE website) remains in effect. The current edition of any standard may be purchased from the ASHRAE Online Store at www.ashrae.org/bookstore or by calling 404-636-8400 or 1-800-727-4723 (for orders in the U.S. or Canada).

The appearance of any technical data or editorial material in this public review document does not constitute endorsement, warranty, or guaranty by ASHRAE of any product, service, process, procedure, or design, and ASHARE expressly disclaims such.

© 2020 ASHRAE. This draft is covered under ASHRAE copyright. Permission to reproduce or redistribute all or any part of this document must be obtained from the ASHRAE Manager of Standards, 1791 Tullie Circle, NE, Atlanta, GA 30329. Phone: 404-636-8400, Ext. 1125. Fax: 404-321-5478. E-mail: standards.section@ashrae.org.

ASHRAE, 1791 Tullie Circle, NE, Atlanta GA 30329-2305
FOREWORD

This update to the 2015 version that was reaffirmed in 2018 includes an update of the steady-state criteria for recording data. This standard meets ASHRAE’s mandatory language requirements.

Selecting an appropriate gas flowmeter can be a daunting task given the wide variety of operating principles, measurement precision, and costs of commercial products. Whether gas flow measurements are to be taken in a laboratory or in the field, selecting the appropriate meter should be based on the required measurement accuracy. Once a gas flowmeter has been selected, the user may need to consult with the meter manufacturer regarding installation specifics, operating range limits, calibration limits, and other similar specifics in order to obtain the expected measurement accuracy. Safety is an important consideration for all procedures involving gases, particularly regarding flammability, toxicity, and corrosiveness – wear safety glasses and other personal protection equipment.

1. PURPOSE

This standard prescribes methods for gas flow measurement.

2. SCOPE

This standard applies to laboratory and field gas flow measurement for testing heating, ventilating, air-conditioning, and refrigerating systems and components. This standard is restricted to applications where the entire flow stream of gas enters and exits the gas flowmeter in a “gas-only” state during data recording with the following exceptions:

a. This standard does not apply to airflow measurements at pressures within this range: -25 kPa to +25 kPa (-100 in. of water to +100 in. of water) referenced to ambient pressure. Those measurements are within the scope of ASHRAE Standard 41.2.

b. This standard does not apply to fan performance rating airflow measurements. Those measurements are within the scope of ASHRAE Standard 51.

c. This standard does not apply to gaseous-phase refrigerant mass flow measurements where the gas flow includes circulating lubricant. Those measurements are within the scope of ASHRAE Standard 41.10.

3. DEFINITIONS

The following definitions apply to the terms used in this standard.

accuracy: the degree of conformity of an indicated value to the corresponding true value.

error: the difference between the test result and its corresponding true value.

genuometrically equivalent diameter: the diameter of a circle having the same area as a rectangular area.

hydraulic diameter: four times the gas flow area divided by the perimeter of the solid boundary in contact
with the gas.

**measurement system:** the instruments, signal conditioning systems if any, and data acquisition system if any.

**operating tolerance limit:** the upper or lower value of an operating tolerance that is associated with a test point or a targeted set point.

**random error:** the portion of the total error that varies randomly in repeated measurements of the *true value* throughout a test process.

**systematic error:** the portion of the total error that remains constant in repeated measurements of the *true value* throughout a test process.

**targeted set point:** a specific set of test conditions where the required gas mass flow rate is known and has an associated operating tolerance.

**test point:** a specific set of test operating conditions for recording data where the measured required gas mass flow rate is unknown and has an associated operating tolerance.

**uncertainty:** a measure of the potential error in a measurement that reflects the lack of confidence in the result to a specified level.

**unit under test (UUT):** equipment that is the subject of gas flow rate measurements.

4. **CLASSIFICATIONS**

4.1 **Gas Flow Operating State.** Gas flow measurement methods shall be restricted to applications where the entire gas flow stream enters and exits the gas flowmeter in the “vapor-only” state during data recording. Trace amounts of liquids shall be less than 1% by mass unless otherwise specified by the flowmeter manufacturer or by the Test Plan in Section 5.1.

4.2 **Gas Flow Measurement Applications.** Gas flow measurement applications that are within the scope of this standard shall be classified as one of the following types:

4.2.1 **Laboratory Applications.** Gas flow measurements under laboratory conditions are engineering development tests or tests to determine product ratings.

**Informative Note:** Laboratory gas flow measurements tend to use more accurate instruments than field measurements, and tend to meet the instrument manufacturer’s installation requirements.

4.2.2 **Field Applications.** Gas flow measurements under field conditions are tests to determine installed system gas flow rates.

**Informative Note:** Field gas flow measurements tend to use less accurate instruments than laboratory measurements, and often do not meet the instrument manufacturer’s installation requirements.

4.3 **Gas Flow Meters**

4.3.1 **Gas Mass Flow Meters.** Gas flow meters in this category perform direct measurement of gas mass flow rates.

4.3.2 **Gas Volumetric Flow Meters.** Gas flow meters in this category perform direct measurement of gas volumetric flow rates. If gas mass flow rates are required, each gas volumetric flow
measurement shall be multiplied by the inlet gas density at the flow measurement location to obtain the gas mass flow rate measurement.

Informative Note: Ultrasonic flowmeters, vortex-shedding flowmeters, and drag-force flowmeters are examples of velocity-measuring devices that can be used to determine volumetric flow rates or gas mass flow rate if the density in the measurement plane is determinable.

4.4 Gas Flow Measurement Methods. Gas flow measurement methods that are within the scope of this standard are the methods listed below. Each of these gas flow measurement methods is described in Section 7.5.

   a. Coriolis flowmeters
   b. Thermal flowmeters
   c. Orifice meters
   d. Flow nozzles
   e. Venturi tubes
   f. Turbine flowmeters
   g. Variable-area flowmeters
   h. Ultrasonic flowmeters
   i. Pitot-static tube methods
   j. Vortex-shedding flowmeters
   k. Drag-force flowmeters.

5. REQUIREMENTS

5.1 Test Plan. A test plan shall specify the gaseous mass flow rate measurement system accuracy. The test plan shall also include the test points, targeted set points, and corresponding operating tolerances to be performed. The test plan shall be one of the following options:

   a. A document provided by the person or the organization that authorized the tests and calculations to be performed.
   c. A rating standard.
   d. A regulation or code.

5.2 Values to be Determined and Reported. The test values to be determined and reported shall be as shown in Table 1. Use the unit of measure in the Table 1 unless otherwise specified in the test plan in Section 5.1.
### Table 1 Measurement Values and Units of Measure

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Units of Measure</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas mass flow rate and uncertainty</td>
<td>kilogram per second (kg/s)</td>
<td>No notes apply.</td>
</tr>
<tr>
<td>Gas volumetric flow rate and uncertainty</td>
<td>cubic meters per second (m³/s)</td>
<td>Only if specified in test plan in Section 5.1.</td>
</tr>
<tr>
<td>Gas density and uncertainty</td>
<td>kilograms per cubic meter (kg/m³)</td>
<td>Only if specified in test plan in Section 5.1.</td>
</tr>
</tbody>
</table>

### 5.3 Test Requirements

#### 5.3.1 Accuracy
A selected gas flowmeter shall meet or exceed the required gas flow measurement system accuracy specified in the test plan in Section 5.1 over the full range of operating conditions.

#### 5.3.2 Uncertainty
The uncertainty in each gas flow measurement shall be calculated using the method in Section 8 for each test point, unless otherwise stated in the test plan in Section 5.1. Alternatively, the worst-case uncertainty for all test points shall be estimated and the same value reported for each test point.

#### 5.3.3 Steady-State Test Criteria for Gas Mass Flow Rate Measurements
Gas mass flow rate test data shall be recorded at steady-state conditions unless otherwise specified in the test plan in Section 5.1. If the test plan requires gas mass flow rate test data points to be recorded at steady-state test conditions and provides the operating condition tolerance but does not specify the steady-state criteria, then determine that steady-state test conditions have been achieved using one of the following methods:

- a. Apply the steady-state criteria in Section 5.3.3.1 if the test plan provides test points for gas mass flow rate measurement.
- b. Apply the steady-state criteria in Section 5.3.3.2 if the test plan provides targeted set points for gas mass flow rate measurement.

#### 5.3.3.1 Steady-State Gas Mass Flow Rate Criteria for Test Points
Starting with the time set to zero, sample not less than 30 gas mass flow rate measurements \( N \) at equal time intervals \( \delta t \) over a test duration \( \Delta t \) where \( \Delta t \) is in time units. Equation 5-1 states the relationship of the test duration to the number of gas mass flow rate samples and the equal time intervals.

\[
\Delta t = (N - 1)\delta t
\]  

**Informative Note:** Circumstances for measurement vary, so the user should select a duration of test and the equal time intervals based upon the longest period of the observed gas mass flow rate fluctuations during operation near the steady-state conditions.

Record each sampled gas mass flow rate measurement \( \dot{m}_i \) and the corresponding time \( t_i \). Apply the least-squares line method to determine the slope \( b \) of the gas mass flow rate data trend line illustrated in Figure 1 using Equation 5-2.

\[
b = \left( \frac{N(\sum_{i=1}^{N} t_i \dot{m}_i) - (\sum_{i=1}^{N} t_i)(\sum_{i=1}^{N} \dot{m}_i)}{N(\sum_{i=1}^{N} t_i^2) - (\sum_{i=1}^{N} t_i)^2} \right)
\]  

**Informative Note:** It should be noted that the units for the slope in Equation 5-2 are gas mass flow rate, kg/s (lbm/min), divided by the units that the user has selected for time.
Determine the mean offset $\mu$ of the sampled data using Equation 5-3, and then calculate the standard deviation $\sigma$ using Equation 5-4.

\[
\mu = \frac{1}{N} \left[ \sum_{i=1}^{N} (\bar{m}_i - bt_i) \right], \text{ kg/s (lbm/min)} \tag{5-3}
\]

\[
\sigma = \left[ \left( \frac{1}{(N-2)} \sum_{i=1}^{N} (\bar{m}_i - bt_i - \mu)^2 \right)^{1/2} \right], \text{ kg/s (lbm/min)} \tag{5-4}
\]

The mean of the sampled gas mass flow rates $\bar{m}$ is defined by Equation 5-5.

\[
\bar{m} = \frac{1}{N} \left[ \sum_{i=1}^{N} (\bar{m}_i) \right], \text{ kg/s (lbm/min)} \tag{5-5}
\]

$\bar{m}$, as determined by Equation 5-5, represents the steady-state mean gas mass flow rate provided that one of the following criteria is satisfied:

a. Apply Equation 5-6 if $2\sigma \geq \bar{m}_L$, where $\bar{m}_L$ is the specified operating tolerance limit for gas mass flow rate, and if Equation 5-6 is satisfied by not less than 95% of the sampled gas mass flow rates.
\[ |\dot{m}_i - \mu| \leq 2\sigma, \text{ kg/s (lbm/min)} \quad (5-6) \]

b. Apply Equation 5-7 if \( \bar{m}_L \geq 2\sigma \) where \( \bar{m}_L \) is the specified operating tolerance limit for gas mass flow rate, and if Equation 5-7 is satisfied by not less than 95\% of the sampled gas mass flow rates.

\[ |\dot{m}_i - \mu| \leq \bar{m}_L, \text{ kg/s (lbm/min)} \quad (5-7) \]

**Informative Note:** For further reading about this method of determining steady-state conditions, refer to Informative Annex A – Bibliography items A1 and A2.

### 5.3.3.2 Steady-State Gas Mass Flow Rate Criteria for Targeted Set Points

Starting with the time set to zero, sample not less than 30 gas mass flow rate measurements \( N \) at equal time intervals \( \delta t \) over a test duration \( \Delta t \) where \( \Delta t \) is in time units. Equation 5-8 states the relationship of the test duration to the number of samples and the equal time intervals.

\[ \Delta t = (N - 1)\delta t \quad (5-8) \]

**Informative Note:** Circumstances for measurement vary, so the user should select a duration of test and the equal time intervals based upon the longest period of the observed gas mass flow rate fluctuations during operation near the steady-state conditions.

Record each sampled gas mass flow rate measurement \( \dot{m}_i \) and the corresponding time \( t_i \). Apply the least-squares line method to determine the slope \( b \) of the gas mass flow rate data trend line illustrated in Figure 2 using Equation 5-9

\[ b = \left( \frac{\left[ N(\sum_{i=1}^{N} t_i \dot{m}_i) - (\sum_{i=1}^{N} t_i)(\sum_{i=1}^{N} \dot{m}_i) \right]}{\left[ N(\sum_{i=1}^{N} t_i^2) - (\sum_{i=1}^{N} t_i)^2 \right]} \right) \quad (5-9) \]

**Informative Note:** It should be noted that the units for the slope in Equation 5-9 are gas mass flow rate, kg/s (lbm/min), divided by the units that the user has selected for time.
Determine the mean offset $\mu$ of the sampled data using Equation 5-10 and calculate the standard deviation $\sigma$ using Equation 5-11.

$$\mu = \frac{1}{N} \left[ \sum_{i=1}^{N} (\bar{m}_i - b t_i) \right], \text{kg/s (lbm/min)} \quad (5-10)$$

$$\sigma = \sqrt{\left( \frac{1}{(N-2)} \sum_{i=1}^{N} (\bar{m}_i - b t_i - \mu)^2 \right)^{\frac{1}{2}}}, \text{kg/s (lbm/min)} \quad (5-11)$$

The mean of the sampled gas mass flow rates, $\bar{m}$, is defined by Equation 5-12.

$$\bar{m} = \frac{1}{N} \left[ \sum_{i=1}^{N} (\bar{m}_i) \right], \text{kg/s (lbm/min)} \quad (5-12)$$

A tolerance on the fluctuations about the trend line represents a limit on the fluctuation level relative to the trend line of the sampled data. If the tolerance of fluctuations about the trend line is not specified in the test plan, the bounds for a 95% confidence limit for the fluctuations about the trend line shall then be determined according to Equation 5-13.

$$|\bar{m} - \bar{m}_{SP}| + |b \Delta t| + 2\sigma \leq \bar{m}_L, \text{kg/s (lbm/min)} \quad (5-13)$$

The steady-state condition of the set point gas mass flow rate, $\bar{m}_{SP}$, exists (a) where Equation 5-14 is satisfied by not less than 95% of the sampled gas mass flow rates where $\bar{m}_L$ is the operating tolerance limit.
for gas mass flow rate

\[(\dot{m}_{SP} - \dot{m}_L) \leq \dot{m}_t \leq (\dot{m}_{SP} + \dot{m}_L), \text{ kg/s (lbm/min)} \quad (5-14)\]

(b) where

\[-0.50 \dot{m}_L \leq (\bar{m} - \dot{m}_{SP}) \leq 0.50 \dot{m}_L, \text{ kg/s (lbm/min)} \quad (5-15)\]

and (c) where

\[b \Delta t \leq 0.50 \dot{m}_L, \text{ kg/s (lbm/min)} \quad (5-16)\]

**Informative Note:** For further reading about this method of determining steady-state conditions, refer to Informative Annex A – Bibliography items A1 and A2.

5.3.4 Unsteady Gas Mass Flow Rate Measurements. If required by the test plan in Section 5.1, gas mass flow rate test data shall be recorded:

a. at operating conditions that are not steady state,
b. at the time intervals specified in the test plan,
c. within the test condition limits specified in the test plan,
d. using instrument response times specified in the test plan.

5.3.5 Steady-State Test Criteria for Gas Volumetric Flow Rate Measurements. Gas volumetric flow rate test data shall be recorded at steady-state conditions if required in the test plan in Section 5.1. If the test plan requires gas volumetric flow rate test data points to be recorded at steady-state test conditions and provides the operating condition tolerance but does not specify the steady-state criteria, then determine that steady-state test conditions have been achieved using one of the following methods:

a. Apply the steady-state criteria in Section 5.3.5.1 if the test plan provides test points for gas volumetric flow rate measurement.
b. Apply the steady-state criteria in Section 5.3.5.2 if the test plan provides targeted set points for gas volumetric flow rate measurement.

5.3.5.1 Steady-State Gas Volumetric Flow Rate Criteria for Test Points

Starting with the time set to zero, sample not less than 30 gas volumetric flow rate measurements \(N\) at equal time intervals \(\delta t\) over a test duration \(\Delta t\) where \(\Delta t\) is in time units. Equation 5-17 states the relationship of the test duration to the number of gas volumetric flow rate samples and the equal time intervals.

\[\Delta t = (N - 1)\delta t \quad (5-17)\]

**Informative Note:** Circumstances for measurement vary, so the user should select a duration of test and the equal time intervals based upon the longest period of the observed gas volumetric flow rate fluctuations during operation near the steady-state conditions.

Record each sampled gas volumetric flow rate measurement \(Q_t\) and the corresponding time \(t_i\). Apply the least-squares line method to determine the slope \(b\) of the gas volumetric flow rate data trend line illustrated in Figure 3 using Equation 5-18.

\[b = \left\{ \frac{N(\sum_{i=1}^{N} t_i Q_t) - (\sum_{i=1}^{N} t_i)(\sum_{i=1}^{N} Q_t)}{N(\sum_{i=1}^{N} t_i^2) - (\sum_{i=1}^{N} t_i)^2} \right\} \quad (5-18)\]

**Informative Note:** It should be noted that the units for the slope in Equation 5-2 are gas
volumetric flow rate, m³/s (cfm), divided by the units that the user has selected for time.

Determine the mean offset $\mu$ of the sampled data using Equation 5-19, and then calculate the standard deviation $\sigma$ using Equation 5-20.

$$\mu = \frac{1}{N} \left[ \sum_{i=1}^{N} (Q_i - bt_i) \right], \text{ m}^3/\text{s (cfm)} \tag{5-19}$$

$$\sigma = \left[ \left( \frac{1}{N-2} \right) \sum_{i=1}^{N} (Q_i - bt_i - \mu)^2 \right]^{1/2}, \text{ m}^3/\text{s (cfm)} \tag{5-20}$$

The mean of the sampled gas volumetric flow rates $\bar{Q}$ is defined by Equation 5-21.

$$\bar{Q} = \frac{1}{N} \left[ \sum_{i=1}^{N} (Q_i) \right], \text{ m}^3/\text{s (cfm)} \tag{5-21}$$

$\bar{Q}$, as determined by Equation 5-21, represents the steady-state mean gas volumetric flow rate provided that one of the following criteria is satisfied:

a. Apply Equation 5-22 if $2\sigma \geq Q_L$, where $Q_L$ is the specified operating tolerance limit for gas volumetric flow rate, and if Equation 5-22 is satisfied by not less than 95% of the sampled gas
volumetric flow rates.

\[ |Q_i - \mu| \leq 2\sigma, \text{ m}^3/\text{s (cfm)} \quad (5-22) \]

b. Apply Equation 5-23 if \( Q_L \geq 2\sigma \) where \( Q_L \) is the specified operating tolerance limit for gas volumetric flow rate, and if Equation 5-23 is satisfied by not less than 95% of the sampled gas volumetric flow rates.

\[ |Q_i - \mu| \leq Q_L, \text{ m}^3/\text{s (cfm)} \quad (5-23) \]

Informative Note: For further reading about this method of determining steady-state conditions, refer to Informative Annex A—Bibliography items A1 and A2.

5.3.5.2 Steady-State Gas Volumetric Flow Rate Criteria for Targeted Set Points

Starting with the time set to zero, sample not less than 30 gas volumetric flow rate measurements \( N \) at equal time intervals \( \delta t \) over a test duration \( \Delta t \) where \( \Delta t \) is in time units. Equation 5-24 states the relationship of the test duration to the number of samples and the equal time intervals.

\[ \Delta t = (N - 1)\delta t \quad (5-24) \]

Informative Note: Circumstances for measurement vary, so the user should select a duration of test and the equal time intervals based upon the longest period of the observed gas volumetric flow rate fluctuations during operation near the steady-state conditions.

Record each sampled gas volumetric flow rate measurement \( Q_i \) and the corresponding time \( t_i \). Apply the least-squares line method to determine the slope \( b \) of the gas volumetric flow rate data trend line illustrated in Figure 4 using Equation 5-25.

\[ b = \frac{\left[ N(\sum_{i=1}^{N} t_i Q_i) - (\sum_{i=1}^{N} t_i)(\sum_{i=1}^{N} Q_i) \right]}{N(\sum_{i=1}^{N} t_i^2) - (\sum_{i=1}^{N} t_i)^2} \quad (5-25) \]

Informative Note: It should be noted that the units for the slope in Equation 5-25 are gas volumetric flow rate, m\(^3\)/s (cfm), divided by the units that the user has selected for time.
Determine the mean offset \( \mu \) of the sampled data using Equation 5-26 and calculate the standard deviation \( \sigma \) using Equation 5-27.

\[
\mu = \frac{1}{N} \left[ \sum_{i=1}^{N} (Q_i - bt_i) \right], \text{ m}^3/\text{s (cfm)} \quad (5-26)
\]

\[
\sigma = \left[ \left( \frac{1}{(N-2)} \right) \sum_{i=1}^{N} (Q_i - bt_i - \mu)^2 \right]^{\frac{1}{2}}, \text{ m}^3/\text{s (cfm)} \quad (5-27)
\]

The mean of the sampled gas volumetric flow rates \( \bar{Q} \) is defined by Equation 5-28.

\[
\bar{Q} = \frac{1}{N} \left[ \sum_{i=1}^{N} (Q_i) \right], \text{ m}^3/\text{s (cfm)} \quad (5-28)
\]

A tolerance on the fluctuations about the trend line represents a limit on the fluctuation level relative to the trend line of the sampled data. If the tolerance of fluctuations about the trend line is not specified in the test plan, the bounds for a 95% confidence limit for the fluctuations about the trend line shall then be determined according to Equation 5-29.

\[
|\bar{Q} - Q_{SP}| + |b\Delta t| + 2\sigma \leq Q_L, \text{ m}^3/\text{s (cfm)} \quad (5-29)
\]

The steady-state condition of the set point gas volumetric flow rate, \( Q_{SP} \), exists (a) where Equation 5-30 is satisfied by not less than 95% of the sampled gas volumetric flow rates where \( Q_L \) is the operating tolerance.
limit for gas volumetric flow rate

\[(Q_{SP} - Q_L) \leq Q_i \leq (Q_{SP} + Q_L), \text{m}^3/\text{s (cfm)}\]  \hspace{0.5cm} (5-30)

(b) where

\[-0.50 Q_L \leq (\bar{Q} - Q_{SP}) \leq 0.50 Q_L, \text{m}^3/\text{s (cfm)}\]  \hspace{0.5cm} (5-31)

and (c) where

\[b\Delta t \leq 0.50Q_L, \text{m}^3/\text{s (cfm)}\]  \hspace{0.5cm} (5-32)

Informative Note: For further reading about this method of determining steady-state conditions, refer to Informative Annex A – Bibliography items A1 and A2.

5.3.6 Unsteady Gas Volumetric Flow Rate Measurements. If required by the test plan in Section 5.1, gas mass flow rate test data or gas volumetric flow rate data shall be recorded:
1. at operating conditions that are not steady state,
2. at the time intervals specified in the test plan,
3. within the test condition limits specified in the test plan,
4. using instrument response times specified in the test plan.

5.3.7 Gas Properties. If not specified in the test plan in Section 5.1, the gas property data shall be obtained from the NIST Standard Reference Database 23 (REFPROP)\(^1\) or from the source of the gas and shall be recorded in the test report.

Informative Note: Informative Annex A Section A1 identifies one potential source of gas properties.

5.3.8 Operating Limits. Operating conditions during gas flow rate data measurements shall not exceed limits for pressure, pressure differential, temperature, gas velocity, or pressure pulsations specified in the test plan or by the gas flowmeter manufacturer to achieve the measurement system accuracy required by the test plan.

5.3.9 Leakage Requirement. Unless otherwise specified in the test plan in Section 5.1, measured gas leakage out of the test apparatus shall be not be greater than 0.25% of the gas flow rate at the greatest pressure tested under laboratory conditions, or not greater than 1% of the gas flow rate at the greatest pressure tested under field conditions.

Informative Note: Account for the leakage in the uncertainty analysis.

5.3.10 Gas Flowmeter Installation. The selected gas flowmeter shall be installed in accordance with instructions from the manufacturer, or the uncertainty calculations shall include estimated uncertainties for installations that are not in accordance with the manufacturer’s instructions.

6. INSTRUMENTS

6.1 Instrumentation Requirements for All Measurements

6.1.1 Instruments and data acquisition systems shall be selected to meet the measurement system accuracy specified in the test plan.
6.1.2 Measurements from the instruments shall be traceable to primary or secondary standards calibrated by the National Institute of Standards and Technology (NIST) or to the Bureau International des Poids et Mesures (BIPM) if a National Metrology Institute (NMI) other than NIST is used. In either case, the indicated corrections shall be applied to meet the uncertainty stated in subsequent sections. Instruments shall be recalibrated on regular intervals that do not exceed the intervals prescribed by the instrument manufacturer and calibration records shall be maintained. Instruments shall be installed in accordance with the instrument manufacturer’s requirements or the manufacturer’s accuracy does not apply.

6.1.3 Instruments shall be applied and used in accordance with the following standards:
   a. Temperature – ASHRAE Standard 41.1 if temperature measurements are required.
   b. Pressure – ASHRAE Standard 41.3 if pressure measurements are required.

6.2 Temperature Measurements

If temperature measurements are required by the test plan in Section 5.1, the measurement system accuracy shall be within the following limits unless otherwise specified in the test plan:
   a. Temperature sensors within ±0.28°C (±0.5°F).
   b. Temperature difference sensors within ±1.0% of the reading.

6.3 Pressure Measurements

6.3.1 Laboratory Pressure Measurements

   6.3.1.1 If pressure measurements are required by the test plan in Section 5.1, the measurement system accuracy shall be within ±1.0% of reading unless otherwise specified in the test plan. If absolute pressure sensors are not used, the barometric pressure shall be added to obtain absolute pressure values prior to performing uncertainty calculations.

   6.3.1.2 If differential pressure measurements are required by the test plan, the measurement system accuracy shall be within ±1.0% of reading unless otherwise specified in the test plan. Pressure shall be measured in close proximity to the flow meter in accordance with the flow meter manufacturer’s specifications.

6.3.2 Field Pressure Measurements

   6.3.2.1 If pressure measurements are required by the test plan in Section 5.1, the measurement system accuracy shall be within ±3.0% of reading unless otherwise specified in the test plan. If absolute pressure sensors are not used, the barometric pressure shall be added to obtain absolute pressure values prior to performing uncertainty calculations.

   6.3.2.2 If differential pressure measurements are required by the test plan, the measurement system accuracy shall be within ±3.0% of reading unless otherwise specified in the test plan. Pressure shall be measured in close proximity to the flow meter in accordance with the flow meter manufacturer’s specifications.

6.4. Time Measurements

Time measurement system accuracy shall be within ±0.5% of the elapsed time measured, including any uncertainty associated with starting and stopping the time measurement unless (a) otherwise specified in the test plan in Section 5.1, or (b) a different value for time measurement system accuracy is required to be consistent with the gas flow rate measurement system accuracy specified in the test plan.
7. GAS FLOW RATE MEASUREMENT METHODS

7.1 Constraint on All Gas Flow Rate Measurement Methods. A selected gas flow measurement plane shall exceed 7.5 geometrically equivalent diameters downstream of an obstruction or any change in the airflow direction and shall exceed 3 geometrically equivalent diameters upstream of an obstruction or change in the gas flow direction unless otherwise specified by the airflow measurement instrument manufacturer. For a rectangular duct with interior width and height dimensions equal to \( a \) and \( b \) respectively, the geometrically equivalent diameter shall be obtained from Equation 7-1. For a round duct, the geometrically equivalent diameter \( D_E \) is equal to the interior diameter \( D \).

\[
D_E = \sqrt{\frac{4ab}{\pi}} \quad (7-1)
\]

where
- \( D_E \) = geometrically equivalent diameter, m (ft)
- \( a \) = interior width, m (ft)
- \( b \) = interior height, m (ft)

7.2 Coriolis Flowmeters. Review Section 7.1. Coriolis gas flowmeters provide direct measurement of gas mass flow rates. In a Coriolis flowmeter, the gas flows through a vibrating sensor tube within the meter. An electromagnetic coil located on the sensor tube vibrates the tube in a cantilever motion at a known frequency. The gas enters a vibrating tube and is given the vertical momentum of the tube. The gas in the entry portion of the sensor tube resists in the downward direction when the tube is moving upward during half of the vibration cycle. Conversely, when the tube is moving downward during half of the vibration cycle, the gas in the exit portion of the sensor tube resists in the upward direction. Combined, these effects create a symmetrical twist angle. According to Newton’s Second Law of Motion, the amount of sensor tube twist angle is directly proportional to the mass flow rate of gas flowing through the tube. Electromagnetic velocity sensors, located on opposing sides of the sensor tube, measure the velocity of the vibrating tube. Mass flow rate is determined by measuring the time difference in the velocity measurements – the greater the time difference, the greater the mass flow rate.

Informative Note: For further reading, see Informative Annex A Section A2.

7.3 Thermal Flowmeters. Review Section 7.1. Thermal flowmeters provide direct measurement of gas mass flow rates. The basic elements of the constant heat input thermal mass flowmeters are two temperature sensors that are positioned on opposite sides of an electric heater. The gas mass flow rate shall be obtained from Equation 7-2.

\[
\dot{m} = \frac{Kq}{C_p(T_2 - T_1)} \quad (7-2)
\]

where
- \( \dot{m} \) = gas mass flow rate, kg/s (lbm/min)
- \( K \) = dimensionless meter coefficient
- \( q \) = electric heat flux rate, kJ/s (Btu/min)
- \( C_p \) = specific heat of the gas, kJ/kg-K (Btu/lb-°F)
- \( T_1 \) = incoming gas temperature, °C (°F)
- \( T_2 \) = outgoing gas temperature, °C (°F)

nozzles, and venturi tubes are mass flow meters. ASME PTC 19.5\textsuperscript{4} and ASME MFC-3M\textsuperscript{5} describe measurement of fluid flow in pipes using orifices, flow nozzles, and venturi tubes, including construction proportions and port locations.

### 7.4.1 Orifices, Flow Nozzles, and Venturi Tube Flowmeter Geometric Profiles.

Figure 5 illustrates the geometric profile of an orifice metering section. Figure 6 illustrates the geometric profile of a long radius nozzle, and Figure 7 shows the geometric profile of a venturi tube.

---

**Figure 5: Orifice Flowmeter Geometric Profile**

Reprinted with Permission of ASME
Figure 6: Long Radius Nozzle Geometric Profile
Reprinted with Permission of ASME

Figure 7: Venturi Tube Flowmeter Geometric Profile
Reprinted with Permission of ASME
7.4.2 Gas Mass Flow Rate Equations and Procedures. This section provides the equations procedures for calculating gas mass flow rates using long radius nozzles and provides reference information for calculating gas mass flow rates for orifices, ISA 1932 nozzles, venturi nozzles, or venturi tubes.

Calculating a gas mass flow rate using these methods requires iteration because (a) the discharge coefficient $C$ is a function of the Reynolds number and the Reynolds number is a function of the average gas flow velocity, and (b) the average gas flow velocity is not known until the gas mass flow rate has been determined. ASME PTC 19.5\(^4\) includes an example of this iterative procedure on page 25, and ASME MFC-3M\(^4\) provides the limits of use, discharge coefficient equations, and expansibility factor equations for orifices, long radius nozzles, ISA 1932 nozzles, venturi nozzles, and venturi tube flowmeters.

7.4.2.1 Measurements: Measurements required for this nozzle gas flow shall be:

a. Inlet duct geometrically equivalent diameter $D_E$ that is defined in Equation 7-1, m (ft)
b. Nozzle throat diameter $d$, m (ft)
c. Nozzle inlet absolute pressure $p_1$, Pa (psia)
d. Nozzle inlet absolute pressure $p_2$, Pa (psia)
e. Nozzle differential pressure $\Delta p = (p_1 - p_2)$, Pa (psia)
f. Nozzle inlet temperature $t_1$, °C (°F)

7.4.2.2 Nozzle Inlet Duct Hydraulic Diameter. Nozzle inlet duct hydraulic diameter $D_h$ shall be obtained from dimensional measurements. For a round duct $D_h$ is equal to the interior inlet diameter. For a rectangular duct, the hydraulic diameter shall be obtained from Equation 7-2.

\[
D_h = \frac{2ab}{(a+b)}
\]

where

$D_h =$ hydraulic diameter, dimensionless
$a =$ interior width, m (ft)
$b =$ interior height, m (ft)

7.4.2.3 Nozzle Limits for Use and Reynolds Number. Limits for the use for long radius nozzle are:

a. $50$ mm (2 in.) $\leq D \leq 630$ mm (25 in.)
b. $R_s/D \leq 3.2 \left(10^4\right)$ where $R_s$ is the mean of the surface roughness in the upstream duct
c. $1 \left(10^4\right) \leq Re_D \leq 1000 \left(10^7\right)$ where $Re_D$ is defined in Equation 7-3.

\[
Re_D = \frac{\rho_1 V D_h}{\mu}
\]

where

$\rho_1 =$ gas density, kg/m\(^3\) (lbm/ft\(^3\))
$V =$ average gas velocity $\left[\frac{4\pi}{\rho_1 \pi D_h^2}\right]$, m/s (ft/s)
$D_h =$ nozzle inlet hydraulic diameter, m (ft)
$\mu =$ dynamic viscosity, Ns/m\(^2\) (lbm/s-ft)

7.4.2.4 Nozzle Beta Ratio. The nozzle beta ratio shall be obtained from Equation 7-4. If gas flow operating temperatures are not within $\pm 6\degree$C ($\pm 10\degree$F) of the ambient temperature during the dimensional measurements, parameters $d$, $D_h$, and $\beta$ shall be corrected to account for thermal expansion in compliance with ASME PTC 19.5\(^7\) Section 3-10.
7.4.2.5 Nozzle Inlet Gas Density. The nozzle inlet gas density $\rho_1$ shall be obtained from the gas property data prescribed in Section 5.3.7 as a function of the nozzle inlet temperature $t_1$ and pressure $p_1$ at each data point.

7.4.2.6 Nozzle Gas Volumetric Flow Rates. Nozzle gas volumetric flow rates shall be calculated from Equation 7-5 in SI units or Equation 7-6 in I-P units.

In SI units:

$$ Q = C\varepsilon \left( \frac{\pi}{4} \right) d^2 K \sqrt{\frac{2(\Delta p)}{\rho_1(1-E\beta^4)}} $$

(7-5)

where

$Q$ = nozzle gas volumetric flow rate, m$^3$/s  
$C$ = nozzle discharge coefficient, dimensionless  
$\varepsilon$ = nozzle expansibility factor, dimensionless  
$d$ = nozzle throat diameter, m  
$K$ = nozzle calibration coefficient, dimensionless  
$\rho_1$ = nozzle inlet gas density, kg/m$^3$  
$\Delta p$ = nozzle differential pressure, Pa  
$E$ = flow kinetic energy coefficient = 1.043$^6$  
$\beta$ = $d/D_h$, dimensionless

In I-P units:

$$ Q = 1097.8C\varepsilon \left( \frac{\pi}{4} \right) d^2 K \sqrt{\frac{(\Delta p)}{\rho_1(1-E\beta^4)}} $$

(7-6)

where

$Q$ = nozzle gas volumetric flow rate, cfm  
$C$ = nozzle discharge coefficient, dimensionless  
$\varepsilon$ = nozzle expansibility factor, dimensionless  
$d$ = nozzle throat diameter, ft  
$K$ = nozzle calibration coefficient, dimensionless  
$\rho_1$ = nozzle inlet gas density, lbm/ft$^3$  
$\Delta p$ = nozzle differential pressure, in. of water  
$E$ = flow kinetic energy coefficient = 1.043$^6$  
$\beta$ = $d/D_h$, dimensionless

7.4.2.7 Nozzle Expansibility Factor. The dimensionless nozzle expansibility factor $\varepsilon$ for a long radius nozzle shall be obtained from Equation 7-7. This equation assumes that the gas is an ideal gas and the gas flow is an isentropic process.

$$ \varepsilon = \left[ \frac{2}{y} \left( \frac{y-1}{y-1} \right) \left( \frac{1-\beta^4}{1-\beta^4} \right) \right]^{1/2} $$

(7-7)

where
7.4.2.8 Nozzle Discharge Coefficient. The dimensionless nozzle discharge coefficient $C$ is a function of $\beta$ and the Reynolds number based upon the nozzle inlet diameter. The discharge coefficient $C$ for long radius nozzles shall be obtained from Equation 7-8.

$$C = 0.9965 - (0.00653\beta^{0.5}) \left(\frac{10^6}{Re_D}\right)^{0.5}$$  \hspace{1cm} (7-8)

The Reynolds number shall be calculated from 7-3, but the average velocity is not known until the gas mass flow rate has been determined. Iteration is required to determine the gas mass flow rate. Choose $C = 1.0$ to begin the iterative calculation procedure for long radius nozzles, ISA 1932 nozzles, venturi nozzles and for venturi tube flowmeters, or choose $C = 0.6$ for orifice flowmeters. Iteration shall continue until the calculated discharge coefficient $C$ matches the previous discharge coefficient within $\pm 0.005$. To calculate gas mass flow rates for orifices, ISA 1932 nozzles, venturi nozzles, or venturi tubes, refer to the paragraphs in ASME MFC-3M5 that are listed in Table 2 and use the same procedures that have been described for the long radius nozzles.

<table>
<thead>
<tr>
<th>Flowmeter Type</th>
<th>Limit of Use Section Number</th>
<th>Discharge Coefficient Equation</th>
<th>Expansibility Factor Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orifices</td>
<td>2-4.3.1</td>
<td>2-4</td>
<td>2-6</td>
</tr>
<tr>
<td>ISA 1932 Nozzles</td>
<td>3-4.1.6.1</td>
<td>3-6</td>
<td>3-7</td>
</tr>
<tr>
<td>Venturi Nozzles</td>
<td>3-4.3.4.1</td>
<td>3-16</td>
<td>3-7</td>
</tr>
<tr>
<td>Venturi Tubes</td>
<td>4-4.5.1</td>
<td>4-4.5.1, 4-5.3</td>
<td>4-3</td>
</tr>
</tbody>
</table>

7.4.2.9 Nozzle Gas Mass Flow Rate. The nozzle gas mass flow rate shall be obtained from Equation 7-9, where $\rho_1$ is the nozzle inlet gas density, $\text{kg/m}^3$ ($\text{lbm/ft}^3$) and $Q$ is the gas volumetric flow rate, $\text{m}^3/\text{s}$ (cfm), using Equation 7-5 in SI units or Equation 7-6 in I-P units.

$$\dot{m} = \rho_1 Q, \text{ kg/s (lbm/min)}$$  \hspace{1cm} (7-9)

7.5 Turbine Flow Meters. Review Section 7.1. Turbine flowmeters are volumetric flow meters that have a turbine rotor suspended on low-friction bearings in the gas stream. The rotational speed of the turbine is a linear function of the average gas velocity, and is therefore a linear function of the volumetric flow rate. Turbine rotation is sensed by one of these methods: (a) reluctance sensors, (b) inductance sensors, (c) capacitance sensors, (d) Hall-effect sensors, or (e) mechanical sensors.

Informative Note: For further reading, see Informative Annex A Section A4.

7.6 Variable-Area Flowmeters. Review Section 7.1. Variable-area flowmeters are volumetric flowmeters. These flowmeters consist of a float that is free to move vertically inside a tapered transparent tube that has a graduated scale as shown in Figure 8. The gas to be metered enters at the narrow bottom end of the tube and moves upward, passing through the annulus formed between the float and the inside wall of the tube.
The position of the float is a balance between the gas pressure forces across the annulus acting upward and gravity acting downward on the float.

**Informative Note:** For further reading, see Informative Annex A Section A5.

![Variable-Area Flowmeter](image)

**Figure 8: Variable-Area Flowmeter**

**7.7 Ultrasonic Flowmeters.** Review Section 7.1. Ultrasonic flowmeters measure gas flow velocity. Clamp-on ultrasonic flow meters measure gas velocity within a pipe or tube without being inserted into the flow stream.

**Informative Notes:**
- a. Immersion-type ultrasonic flowmeters are also available.
- b. For further reading, see Informative Annex A Section A3.
- c. Some ultrasonic flowmeters report a volumetric flow rate that is based on a specific pipe diameter.

Ultrasonic flowmeters use the transit-time method to measure the effects that flow velocity has on bi-directional acoustical signals. An upstream transducer sends a signal to a downstream transducer that then returns a signal. When there is no flow, the time for the signal to go from one transducer to other, in either direction, is constant. When gas flow exists, the velocity causes the acoustical signal to increase speed in the direction of flow and reduces the acoustical signal speed in the upstream direction. This creates the time difference that correlates to the flow velocity.

**7.8 Pitot-Static Tube Gas Flow Measurement Methods.** Review Section 7.1. Figure 9 shows an example Pitot-static tube construction and the connections to manometers or electronic pressure transducers. Sections 7.8.1, 7.8.2, and 7.8.3 describe three different methods to determine gas velocity at measurement points in a gas stream by measuring total and static pressures. Pitot-static tubes shall be aligned within ±10 degrees of the gas flow direction, and any misalignment shall be included in the uncertainty estimate.

**Informative Notes:**
- a. Negative pressure readings are a clear indication that the Pitot-static tube is not properly aligned with the direction of gas flow.
b. The presence of significant thermal gradients leads to significant reductions in accuracies unless temperature is measured at each measurement location.

![An Example of a Pitot-Static Tube](image)

**FIGURE 9: An Example of a Pitot-Static Tube**

**7.8.1 Pitot-Static Tube Traverse Gas Flow Measurement.** The process of sequentially positioning a Pitot-static tube at different measuring points within a duct cross section to measure gas velocities is called a Pitot-static tube traverse. The traverse measuring points for a round duct or rectangular duct shall be in accordance with Figure 10.
7.8.1.1 Velocity Pressure. The total pressure $P_t$ is the sum of the static pressure $P_s$, and the velocity pressure $P_v$ so it follows that

$$P_v = P_t - P_s, \text{ Pa (in. of water)} \quad (7-10)$$

7.8.1.2 Average Velocity Pressure. The average velocity pressure $P_{va}$ shall be obtained from Equation 7-11 where $N$ is the number of velocity pressure sampling points

$$P_{va} = \left( \frac{\sum_{i=1}^{N} P_{v,i}}{N} \right)^2, \text{ Pa (in. of water)} \quad (7-11)$$

7.8.1.3 Average Gas Velocity. The average gas velocity shall be obtained from the density at the traverse plane and the average velocity pressure from Equation 7-12 in SI units and from Equation 7-13 in I-P units.

In SI units:

$$V_a = K \sqrt{\frac{2P_{va}}{\rho}} \quad (7-12)$$

where

- $V_a$ = average gas velocity, m/s
- $K$ = calibration coefficient, dimensionless
- $P_{va}$ = pressure, Pa
ρ = gas density in the measurement plane, kg/m³

In I-P units:

\[ V_a = 1097.8K \sqrt{\frac{P_{va}}{\rho}} \]  \hspace{1cm} (7-13)

where

\( V_a \) = average gas velocity, ft/min
\( K \) = calibration coefficient, dimensionless
\( P_{va} \) = pressure, in. of water
\( \rho \) = gas density in the measurement plane, lbm/ft³

7.8.1.4 Gas Volumetric Flow. The gas volumetric flow at the Pitot-static tube traverse plane shall be obtained from Equation 7-14.

\[ Q = V_a A \]  \hspace{1cm} (7-14)

where

\( Q \) = gas volumetric flow rate, m³/s (cfm)
\( V_a \) = average gas velocity, m/s (ft/min)
\( A \) = measurement plane cross section area, m² (ft²)

7.8.2 Self-Averaging Array Gas Flow Measurement. Self-averaging arrays consist of multiple bifurcated or extruded tubes spread out over a measurement plane that have holes to sample and self-average both total and static pressure across the measurement plane. The self-averaged total pressure is connected to one side of a differential pressure transducer, and the self-averaged static pressure is connected to the other side of the same pressure transducer.

7.8.2.1 Average Velocity Pressure. The average velocity pressure shall be obtained from Equation 7-15.

\[ P_{va} = P_{ta} - P_{sa} \]  \hspace{1cm} (7-15)

where

\( P_{va} \) = average velocity pressure, Pa (in. of water)
\( P_{ta} \) = measured average total pressure, Pa (in. of water)
\( P_{sa} \) = measured average static pressure, Pa (in. of water)

7.8.2.2 Average Gas Velocity. The average gas velocity shall be obtained from Equation 7-16 in SI units or from Equation 7-17 in I-P units:

In SI units:

\[ V_a = K \sqrt{\frac{2P_{va}}{\rho}} \]  \hspace{1cm} (7-16)

where

\( V_a \) = average gas velocity, m/s
\( K \) = calibration coefficient, dimensionless
\( P_{va} \) = pressure, Pa
\( \rho \) = gas density in the measurement plane, kg/m³
In I-P units:

\[
V_a = \frac{1097.8K}{\sqrt{P_{va}}} 
\]  

(7-17)

where

\[ V_a \] = average gas velocity, ft/min  
\[ K \] = calibration coefficient, dimensionless  
\[ P_{va} \] = pressure, in. of water  
\[ \rho \] = gas density in the measurement plane, lbm/ft³

7.8.2.3 Gas Volumetric Flow. The gas volumetric flow at the Pitot-static tube array measurement plane shall be obtained from Equation 7-18.

\[
Q = V_a A 
\]  

(7-18)

where

\[ Q \] = gas volumetric flow rate, m³/s [cfm]  
\[ V_a \] = average gas velocity, m/s [ft/min]  
\[ A \] = measurement plane cross section area, m² [ft²]

7.8.3 Self-Averaging Probe Gas Flow Measurement. Self-averaging probes include multiple total and static pressure ports along a straight line or around a circumference within the airstream. The self-averaged total pressure is connected to one side of a differential pressure transducer, and the self-averaged static pressure is connected to the other side of the same pressure transducer.

Informative Note: For further reading, see Informative Annex A Section A7.

7.8.3.1 Average Velocity Pressure. The average velocity pressure shall be obtained from Equation 7-19.

\[
P_{va} = P_{ta} - P_{sa} 
\]  

(7-19)

where

\[ P_{va} \] = average velocity pressure, Pa (in. of water)  
\[ P_{ta} \] = measured average total pressure, Pa (in. of water)  
\[ P_{sa} \] = measured average static pressure, Pa in. (in. of water)

7.8.3.2 Average Gas Velocity. The average gas velocity shall be obtained from Equation 7-20 in SI units or from Equation 7-21 in I-P units:

In SI units:

\[
V_a = K \sqrt[2]{ \frac{P_{va}}{\rho} } 
\]  

(7-20)

where

\[ V_a \] = average gas velocity, m/s  
\[ K \] = calibration coefficient, dimensionless  
\[ P_{va} \] = pressure, Pa  
\[ \rho \] = gas density, kg/m³
In I-P units:

\[ V_a = 1097.8K \sqrt{ \frac{P_{va}}{\rho} } \quad (7-21) \]

where

- \( V_a \) = average gas velocity, ft/min
- \( K \) = calibration coefficient, dimensionless
- \( P_{va} \) = pressure, in. of water
- \( \rho \) = gas density in the measurement plane, lbm/ft³

**7.8.3.3 Gas Volumetric Flow.** The gas volumetric flow at the Pitot-static tube array measurement plane shall be obtained from Equation 7-22.

\[ Q = V_a A \quad (7-22) \]

where

- \( Q \) = gas volumetric flow rate, m³/s [cfm]
- \( V_a \) = average gas velocity, m/s [ft/min]
- \( A \) = measurement plane cross section area, m² [ft²]

**7.9 Vortex-Shedding Flowmeters.** Review Section 7.1. Vortex-shedding flowmeters are used to determine gas velocities. Piezoelectric methods, strain-gage methods, or hot-film methods are used to sense dynamic pressure variations created by vortex shedding. The operating principle for these flowmeters is based on vortex shedding that occurs downstream of an immersed blunt-shaped solid body. As the gas stream passes a blunt-shaped body, the gas separates and generates small vortices that are shed alternately along and downstream of each side of the blunt-shaped body. Each vortex-shedding meter is designed to have a known relationship between the Strouhal number and the Reynolds number so that the vortex shedding frequency is a known function of the gas flow velocity over a specified flow velocity range.

**Informative Note:** For further reading, see Informative Annex A Section A6.

**7.10 Drag-Force Velocity Flowmeters.** Review Section 7.1. Drag-force flowmeters determine gas velocity. Piezoelectric or strain-gage methods are used to sense dynamic drag-force variations. Gas velocity shall be obtained from Equation 7-23 in SI units, or obtained from Equation 7-24 in I-P units.

**Informative Note:** For further reading, see Informative Annex A Section A4.

In SI units:

\[ V = \frac{2 f_d}{\sqrt{C_d \rho}} \quad (7-23) \]

where

- \( V \) = calculated gas velocity, m/s
- \( f_d \) = measured drag force, N
- \( C_d \) = drag coefficient specified by the meter manufacturer, dimensionless
- \( A \) = cross-section area, m²
- \( \rho \) = gas density in the measurement plane, kg/m³

In I-P units:

\[ V = \frac{2 \rho f_d}{\sqrt{C_d \rho}} \quad (7-24) \]
where

\[ V = \text{calculated gas velocity, ft/s} \]
\[ f_d = \text{measured drag force, lb} \]
\[ C_d = \text{drag coefficient specified by the meter manufacturer, dimensionless} \]
\[ A = \text{cross-section area, ft}^2 \]
\[ \rho = \text{gas density in the measurement plane, lb}_m/ft^3 \]
\[ g_c = \text{gravitational constant, 32.174 (lb}_m\text{-ft)/(lb}_f\text{-s}^2) \]

8. UNCERTAINTY REQUIREMENTS.

8.1 Uncertainty Estimate. An estimate of the measurement uncertainty performed in accordance with ASME PTC 19.1\(^6\) shall accompany each gas flow measurement.

Informative Note: Informative Annex B contains an example of uncertainty calculations.

8.2 Method to Express Uncertainty. Assumptions, parameters, and calculations used in estimating uncertainty shall be clearly documented prior to expressing and uncertainty values. Uncertainty shall be expressed as shown in Equation 8-1.

\[ v = \bar{X}_m \pm U_\bar{X} (P\%) \]  \hspace{1cm} (8-1)

where:

- \( v \) = the variable that is a measurement or a calculated result
- \( \bar{X}_m \) = the best estimate of the true value
- \( U_\bar{X} \) = the uncertainty estimate for the variable
- \( P \) = the confidence level, percent

Informative Note: For example: gas mass flow rate = 2.538 kg/s ± 0.013 kg/s (335.7 lbm/min ± 1.7 lbm/min); 95% states that the measured gas flow is believed to be 2.538 kg/s (335.7 lbm/min) with a 95% probability that the true value lies within ±0.013 kg/s (±1.7 lbm/min) of this value.

9. TEST REPORT

If the test plan in Section 5.1 defines the test report requirements, the test report requirements in the test plan supersedes all of the requirements in Section 9. Otherwise, Section 9 specifies the test report requirements.

9.1 Test Identification

a. Date, place, and time.
b. Operator.

9.2 Unit Under Test Description

a. Model number and serial number.
b. Gas specification.
c. Source of gas properties.

9.3 Instrument Description

a. Flowmeter description, model number, serial number, and location within the UUT.
b. Operating range.
c. Instrument accuracy based on specifications or calibration.
d. Documentational evidence of instrument calibrations.

9.4 Measurement System Description
a. Description of instrument installation specifics.
b. Measurement system accuracy based on specifications or calibration.
c. Documentational evidence of instrument calibrations.

9.5 Test Conditions
a. Test conditions in accordance with the test plan in Section 5.1
b. Ambient temperature, °C (°F).
c. Barometric pressure Pa, (psia) if pressures instruments are measuring gauge pressure.

9.6 Test Results

9.6.1 Gas mass flow rate unless otherwise specified by the test plan:
a. Gas mass flow rate, kg/s (lbm/min).
b. Uncertainty in gas mass flow rate, kg/s (lbm/min).

9.6.2 Gas volumetric flow rate if required by the test plan:
a. Gas volumetric flow rate, m³/s (cfm).
b. Uncertainty in gas volumetric flow rate, m³/s (cfm).

9.6.3 Density if required by the test plan:
a. Density, kg/m³ (lbm/ft³)
b. Uncertainty in density, kg/m³ (lbm/ft³)

10. REFERENCES

3. ANSI/ASHRAE Standard 41.3-2014, Standard Methods for Pressure Measurement. ASHRAE, Atlanta, GA. See Note 2

Note 1: Reference 1 is not required if there are no temperature measurements.
Note 2: Reference 2 is not required if there are no pressure measurements.
Note 3: References 3, 4, 5, and 6 are only required if using an Orifice, Flow Nozzle, or Venturi Tube.
(This annex is not part of this standard. It is merely informative and does not contain requirements necessary for conformance to the standard. It has not been processed according to the ANSI requirements for a standard and may contain material that has not been subject to public review or a consensus process. Unresolved objectors on informative material are not offered the right to appeal at ASHRAE or ANSI.)

INFORMATIVE ANNEX A: BIBLIOGRAPHY


INFORMATIVE ANNEX B: AN UNCERTAINTY ANALYSIS EXAMPLE FOR A CORIOLIS FLOWMETER

A Coriolis gas flowmeter is installed in accordance with the flowmeter manufacturer's instructions. The output electronics are connected to a frequency meter that is read automatically by a computer-based data acquisition system. This setup will be used to measure various gas flow mass flow rates from 5% to 100% of the full-scale output. Determine the expected measurement uncertainty at 100% of the full-scale reading, and determine the worst-case uncertainty over the expected operating range.

Follow the step-by-step procedures outlined in Section 9 of ASME PTC 19.1\(^7\), to estimate the uncertainty in SI units in Section B1 or in I-P units in Section B2. Note that, in general, using a commercial equation solver software significantly reduces the time and effort required to complete an uncertainty analysis.

B1. Estimate Uncertainty in SI Units

B1.1. Define the Measurement Process

B1.1.1 Review the test objectives and duration. The test objectives are clearly stated in the description above.

B1.1.2 List all independent measurement parameters and their nominal levels. The only independent measurement is the frequency output from the Coriolis flowmeter. The full-scale output of the flowmeter was set by the manufacturer to 4.0 kg/s, so the nominal flow at 5% of the range is 0.2 kg/s.

B1.1.3 List all calibrations and instrumentation setups that will affect each parameter. The manufacturer verified basic flowmeter operation on their test facility that has a stated uncertainty, $U_{RSS}$, of ±0.05% per ISO 5168\(^{10}\). The calibration data provided when the meter was initially delivered is shown in Table B-1 below. The calibration was a water calibration that uses the total weight collected at a given time period to determine a total flow rate.

<table>
<thead>
<tr>
<th>Flow (%)</th>
<th>Nominal Mass Flow Rate (kg/s)</th>
<th>Meter Total (kg)</th>
<th>Scale Total (kg)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>4.000</td>
<td>307.64</td>
<td>307.57</td>
<td>0.023</td>
</tr>
<tr>
<td>50</td>
<td>2.000</td>
<td>306.44</td>
<td>305.67</td>
<td>0.250</td>
</tr>
<tr>
<td>25</td>
<td>1.000</td>
<td>304.89</td>
<td>305.07</td>
<td>-0.059</td>
</tr>
<tr>
<td>100</td>
<td>4.000</td>
<td>305.04</td>
<td>304.97</td>
<td>0.022</td>
</tr>
</tbody>
</table>
Items to note from the Table B-1:
- The calibration data is in terms of mass only, not mass per unit time. The manufacturer states that the uncertainty related to determining time in the calibration is insignificant.
- Frequency output data from the meter's electronics were not provided; the manufacturer stated that the frequency output is identical to the electronic internal digital value.
- Hysteresis is not well defined – only the 100% flow rate has a repeated point.
- The minimum flow rate measured was 25% due to testing limitation of the meter's manufacturer.
- Because the meter's performance at 5% needs to be determined, this existing data will need to be extrapolated or, preferably, an additional calibration will be required to better define the low-flow performance. Extrapolation at low flows on Coriolis flowmeters should not be done as the error typically increases exponentially.

Figure B-1 shows a typical accuracy estimate from the flowmeter manufacturer with the actual calibrated flow points plotted. From Figure B-1, the expected error will be around 2.5% at a 5% flow rate condition. Because the actual meter’s data appears to be much better than nominal manufacturer's data, an extended calibration should give results with a significantly lower error. Another calibration was performed at an independent calibration lab that also had a stated uncertainty, $U_{RSS}$, of ±0.05%, and the error was determined to be -1.13% at 0.2 kg/s flow rate.

![Figure B-1](image)

**B1.1.** Define the functional relationship between the independent measurement parameters and the test results. Because the mass flow is a direct measurement, there is no functional relationship between multiple measurements and the final test result.

**B1.2.** List Elemental Error Sources

**B1.2.1** Make a complete list of all possible measurement error sources. The number of possible error sources for this system is small due to the simplicity of the overall system. Measurement error sources may include the manufacturer's calibration results, frequency meter measuring error, and data reduction errors in converting from frequency to flow rate.

**B1.2.2** Group the error sources according to the following categories:

**B1.2.2.1 Calibration Errors.** The meter calibration was provided with an uncertainty estimate, so elemental error hierarchy does not need to be estimated for each possible error source.

**B1.2.2.2 Data Acquisition Errors.** Data acquisition errors for the frequency meter include those from the meter calibration that was done at an accredited calibration lab. The meter will be operated in an
ambient temperature that is outside of the instrument’s nominal operating range so temperature effects should be considered.

The communications between the frequency meter and the computer are digital, and it is assumed there is no error associated with transferring these values. This is a logical assumption in this case because a computer error would have been generated if there were a problem in the transmission.

**B1.2.2.3 Data Reduction Errors.** Data reduction errors include the inaccuracy between the curve fit that is used to convert the measured frequency to the flow rate in engineering units.

### B1.3 Calculate Systematic and Random Uncertainty of Parameters

**B1.3.1 Obtain an estimate of each error identified in Section B1.2.**

**B1.3.2 If data is available to estimate the precision index, tentatively classify the error as a precision error. Otherwise, classify it as a bias error.**

### B1.4 Calibration Error Estimate

The manufacturer stated that the flowmeter test facility calibration error was 0.05%. Upon further investigation of their calibration uncertainty analysis, the error appears to be mainly consisting of a bias error that can be expressed as:

\[
B_{\text{Calibration Error \#1}} = 0.035\% \quad S_{\text{Calibration Error \#1}} = 0.015\% \quad \text{Degrees of Freedom} > 30
\]

Using these values for the four calibration points that were reported, Table B-2 gives an estimate of error as a function of flow rate. The entire error is attributed to bias as there is insufficient data available to estimate the precision component.

<table>
<thead>
<tr>
<th>Mass Flow Rate (kg/s)</th>
<th>0.200</th>
<th>1.000</th>
<th>2.000</th>
<th>4.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B_{\text{Calibration Error #2}})</td>
<td>-1.13%</td>
<td>-0.06%</td>
<td>0.25%</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

### B1.5 Data Acquisition Error Estimate

The frequency meter that will be used was specified by the manufacturer to be ±0.010% of the reading for readings greater than 40 Hz. Because the inaccuracy in determining frequency was only specified as a percent reading and did not include a separate term for percent of the measurement range, this error will be assumed to be only a bias error:

\[
B_{\text{Data Acquisition Error \#1}} = 0.010\% \quad S_{\text{Data Acquisition Error \#1}} = 0.000\%
\]

The meter also has a temperature coefficient of ±0.001% per °C for temperatures greater than 28°C. This needs to be considered because the instrumentation console where the meter is located will reach a maximum temperature of 40°C. Again, because the inaccuracy specification did not break up the error into precision and bias components, the error will be assumed to have only a bias component:

\[
B_{\text{Data Acquisition Error \#2}} = 0.001\% \times (40 - 28) = 0.012\% \quad S_{\text{Data Acquisition Error \#2}} = 0.000\%
\]

### B1.6 Data Reduction Error Estimate
The mass flow will be a function related to the measured frequency as shown in the first two columns of Table B-3. Various linear curve fits were reviewed to determine which had the smallest error over the entire operating range. The manufacturer states it is a simple linear relationship that is forced to go through zero and that is how the transmitter was set up. The flow and error associated using this curve fit is shown in the next two columns in Table B-3. The next two columns are the results of a least-square curve fit that includes a point at zero flow rate. The last two columns are the results of a least-square curve fit that excludes the point at zero flow rate. Based on the results in Table B-3, it was decided to use the No Zero Flow Curve Fit. All of this error will be a bias error because it is fixed for a given frequency input in the equation.

### TABLE B-3 Data Reduction Curve Fit Error Comparison

<table>
<thead>
<tr>
<th>Calibration Hertz</th>
<th>Calibration Transmitter Curve Fit</th>
<th>Zero Flow Curve Fit</th>
<th>No Zero Flow Curve Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured Flow, kg/s</td>
<td>Calculated Flow, kg/s</td>
<td>% Flow Error</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.00</td>
<td>-0.00049</td>
</tr>
<tr>
<td>100</td>
<td>0.19777</td>
<td>0.20</td>
<td>-1.130%</td>
</tr>
<tr>
<td>500</td>
<td>0.9994</td>
<td>1.00</td>
<td>-0.060%</td>
</tr>
<tr>
<td>1000</td>
<td>2.0050</td>
<td>2.00</td>
<td>0.249%</td>
</tr>
<tr>
<td>2000</td>
<td>4.0009</td>
<td>4.00</td>
<td>0.022%</td>
</tr>
</tbody>
</table>

Linear Curve Fit Coefficients \((y = mx + b)\) where \(y = \text{Measured Flow} \) and \(x = \text{Output}\)

<table>
<thead>
<tr>
<th>(m)</th>
<th>0.00200000</th>
<th>0.0020015</th>
<th>0.002002</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>0.00000000</td>
<td>-0.0004853</td>
<td>-0.000801</td>
</tr>
</tbody>
</table>

**B1.7 Calculate the Bias and Precision Errors for Each Parameter.** The results from the previously defined elements are now summarized at each of the four calibrated flow rates in Table B-4. The summing of the terms is by the Root-Sum-Square method because a 95% confidence level is desired.
### TABLE B-4 Bias and Precision Error Summary

<table>
<thead>
<tr>
<th>Flow Rate (kg/s)</th>
<th>% BIAS ERRORS</th>
<th>PRECISION ERRORS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calibration Facility</td>
<td>Transmitter &amp;/or Application</td>
</tr>
<tr>
<td></td>
<td>#1</td>
<td>#2</td>
</tr>
<tr>
<td>0.200</td>
<td>0.035%</td>
<td>-1.130%</td>
</tr>
<tr>
<td>1.000</td>
<td>0.035%</td>
<td>-0.060%</td>
</tr>
<tr>
<td>2.000</td>
<td>0.035%</td>
<td>0.249%</td>
</tr>
<tr>
<td>4.000</td>
<td>0.035%</td>
<td>0.022%</td>
</tr>
<tr>
<td>Data Acquisition</td>
<td>#1</td>
<td>#2</td>
</tr>
<tr>
<td></td>
<td>0.01%</td>
<td>0.012%</td>
</tr>
<tr>
<td>Data Reduction</td>
<td>0.200</td>
<td>-0.813%</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>-0.067%</td>
</tr>
<tr>
<td></td>
<td>2.000</td>
<td>0.203%</td>
</tr>
<tr>
<td></td>
<td>4.000</td>
<td>-0.044%</td>
</tr>
</tbody>
</table>

**B1.8 Propagate the Bias and Precision Errors**

**B1.8.1** The bias and precision errors of the independent parameters are propagated separately all the way to the final test result. The individual terms are now summed together again by the Root-Sum-Square method because a 95% confidence level is desired as shown in the second and third columns of Table B-4.

**B1.8.2** Error propagation is done, according to the functional relationship of (a)(4) above, via a Taylor series. This requires a calculation of the sensitivity factors, either by differentiation or by computer perturbation. Because the mass flow rate is the only independent parameter, this step is not required. The values determined in the last step are the ones that will be used in the next step to calculate uncertainty.

**B1.9 Calculate Uncertainty**

Select $U_{ADD}$ or $U_{RSS}$ as models for combining the bias and precision errors of the test result and obtain the uncertainty. $U_{RSS}$ will be used because a 95% confidence level is desired. The value of $t$ for the Student’s $t$ function is taken to be 2.00 because the degrees of freedom for the meter calibration is greater than 30. The final estimate is shown in the fourth column of Table B-5.
TABLE B-5  Propagation and Final Uncertainty Estimate

<table>
<thead>
<tr>
<th>Flow Rate (kg/s)</th>
<th>PROPAGATED ERRORS</th>
<th>95% Confidence Uncertainty Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BIAS</td>
<td>PRECISION</td>
</tr>
<tr>
<td>0.200</td>
<td>1.393%</td>
<td>0.015%</td>
</tr>
<tr>
<td>1.000</td>
<td>0.098%</td>
<td>0.015%</td>
</tr>
<tr>
<td>2.000</td>
<td>0.324%</td>
<td>0.015%</td>
</tr>
<tr>
<td>4.000</td>
<td>0.063%</td>
<td>0.015%</td>
</tr>
</tbody>
</table>

B1.10 Report

B1.10.1 The report summary shall contain the nominal level of the test result, bias limit, precision index, degrees of freedom, and uncertainty of the test result, stating the model used.

B1.10.2 The report shall include a table of the elemental errors included in the uncertainty analysis along with the bias limit, precision index, and degrees of freedom of each parameter. The report for this analysis would include the average value determined per Section 5.2.1 at the actual test condition and the appropriate values from Tables B-4 and B-5.

B2. Estimate Uncertainty in I-P Units

B2.1 Define the Measurement Process

B2.1.1 Review the test objectives and duration. The test objectives are clearly stated in the description above.

B2.1.2 List all independent measurement parameters and their nominal levels. The only independent measurement is the frequency output from the Coriolis flowmeter. The full-scale output of the flowmeter was set by the manufacturer to 31760 lbm/h, so the nominal flow at 5% of the range is 1588 lbm/h.

B2.1.3 List all calibrations and instrumentation setups that will affect each parameter. The manufacturer verified basic flowmeter operation on their test facility that has a stated uncertainty, U_RSS, of ±0.05% per ISO 5168^10. The calibration data provided when the meter was initially delivered is shown in Table B-6 below. The calibration was a water calibration that uses the total weight collected at a given time period to determine a total flow rate.

TABLE B-6 Manufacturer’s Initial Flowmeter Calibration Data

<table>
<thead>
<tr>
<th>Flow Rate (%)</th>
<th>Nominal Flow Rate (lbm/h)</th>
<th>Meter Total (lbm)</th>
<th>Scale Total (lbm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>31760</td>
<td>678.35</td>
<td>678.19</td>
<td>0.023</td>
</tr>
<tr>
<td>50</td>
<td>15880</td>
<td>675.70</td>
<td>674.00</td>
<td>0.250</td>
</tr>
<tr>
<td>25</td>
<td>7940</td>
<td>672.28</td>
<td>672.68</td>
<td>-0.059</td>
</tr>
<tr>
<td>100</td>
<td>31760</td>
<td>672.61</td>
<td>672.46</td>
<td>0.022</td>
</tr>
</tbody>
</table>
Items to note from the Table B-6:
- The calibration data is in terms of mass only, not mass per unit time. The manufacturer states that the uncertainty related to determining time in the calibration is insignificant.
- Frequency output data from the meter's electronics were not provided; the manufacturer stated that the frequency output is identical to the electronic internal digital value.
- Hysteresis is not well defined – only the 100% flow rate has a repeated point.
- The minimum flow rate measured was 25% due to testing limitation of the meter's manufacturer.
- Because the meter's performance at 5% needs to be determined, this existing data will need to be extrapolated or, preferably, an additional calibration will be required to better define the low-flow performance. Extrapolation at low flows on Coriolis flowmeters should not be done as the error typically increases exponentially.

Figure B-2 shows a typical accuracy estimate from the flowmeter manufacturer with the actual calibrated flow points plotted. From Figure B-1, the expected error will be around 2.5% at a 5% flow rate condition. Because the actual meter’s data appears to be much better than nominal manufacturer's data, an extended calibration should give results with a significantly lower error. Another calibration was performed at an independent calibration lab that also had a stated uncertainty, $U_{RSS}$, of ±0.05%, and the error was determined to be -1.13% at 0.44 lbm/s flow rate.

![FIGURE B-2](image)

**FIGURE B-2**
Flow Error vs. Flow Rate

---

**B2.1.4** Define the functional relationship between the independent measurement parameters and the test results. Because the mass flow is a direct measurement, there is no functional relationship between multiple measurements and the final test result.

**B2.2** List Elemental Error Sources

**B2.2.1** Make a complete list of all possible measurement error sources. The number of possible error sources for this system is small due to the simplicity of the overall system. Measurement error sources may include the manufacturer's calibration results, frequency meter measuring error, and data reduction errors in converting from frequency to flow rate.

**B2.2.2** Group the error sources according to the following categories:

**B2.2.2.1** Calibration Errors. The meter calibration was provided with an uncertainty estimate, so elemental error hierarchy does not need to be estimated for each possible error source.

**B2.2.2.2** Data Acquisition Errors. Data acquisition errors for the frequency meter include those from the meter calibration that was done at an accredited calibration lab. The meter will be operated in an
ambient temperature that is outside of the instrument’s nominal operating range, so temperature effects should be considered.

The communications between the frequency meter and the computer are digital, and it is assumed there is no error associated with transferring these values. This is a logical assumption in this case because a computer error would have been generated if there were a problem in the transmission.

**B2.2.2.3 Data Reduction Errors.** Data reduction errors include the inaccuracy between the curve fit that is used to convert the measured frequency to the flow rate in engineering units.

**B2.3 Calculate Systematic and Random Uncertainty of Parameters**

**B2.3.1 Obtain an estimate of each error identified in Section B2.2.**

**B2.3.2 If data is available to estimate the precision index, tentatively classify the error as a precision error. Otherwise, classify it as a bias error.**

**B2.4 Calibration Error Estimate**

The manufacturer stated that the flowmeter test facility calibration error was 0.05%. Upon further investigation of their calibration uncertainty analysis, the error appears to be mainly consisting of a bias error that can be expressed as:

\[ B_{\text{Calibration Error #1}} = 0.035\% \quad S_{\text{Calibration Error #1}} = 0.015\% \quad \text{Degrees of Freedom} >30 \]

Using these values for the four calibration points that were reported, Table B-7 gives an estimate of error as a function of flow rate. The entire error is attributed to bias as there is insufficient data available to estimate the precision component.

<table>
<thead>
<tr>
<th>Mass Flow Rate (lbm/h)</th>
<th>1588</th>
<th>7940</th>
<th>15880</th>
<th>31760</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_{\text{Calibration Error #2}} )</td>
<td>-1.13%</td>
<td>-0.06%</td>
<td>0.25%</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

**B2.5 Data Acquisition Error Estimate**

The frequency meter that will be used was specified by the manufacturer to be ±0.010% of the reading for readings greater than 40 Hz. Because the inaccuracy in determining frequency was only specified as a percent reading and did not include a separate term for percent of the measurement range, this error will be assumed to be only a bias error:

\[ B_{\text{Data Acquisition Error #1}} = 0.010\% \quad S_{\text{Data Acquisition Error #1}} = 0.000\% \]

The meter also has a temperature coefficient of ±0.002% per °F for temperatures greater than 82°F. This needs to be considered because the instrumentation console where the meter is located will reach a maximum temperature of 104°F. Again, because the inaccuracy specification did not break up the error into precision and bias components, the error will be assumed to have only a bias component:

\[ B_{\text{Data Acquisition Error #2}} = 0.001\% \times (40 - 28) = 0.012\% \quad S_{\text{Data Acquisition Error #2}} = 0.000\% \]

**B2.6 Data Reduction Error Estimate**
The mass flow will be a function related to the measured frequency as shown in the first two columns of Table B-8. Various linear curve fits were reviewed to determine which had the smallest error over the entire operating range. The manufacturer states it is a simple linear relationship that is forced to go through zero and that is how the transmitter was set up. The flow and error associated using this curve fit is shown in the next two columns in Table B-8. The next two columns are the results of a least-square curve fit that includes a point at zero flow rate. The last two columns are the results of a least-square curve fit that excludes the point at zero flow rate. Based on the results in Table B-8, it was decided to use the No Zero Flow Curve Fit. All of this error will be a bias error because it is fixed for a given frequency input in the equation.

<table>
<thead>
<tr>
<th>Calibration Output Hertz</th>
<th>Measured Flow, lbm/h</th>
<th>Transmitter Curve Fit</th>
<th>Zero Flow Curve Fit</th>
<th>No Zero Flow Curve Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Calculated Flow, lbm/h</td>
<td>% Flow Error</td>
<td>Calculated Flow, lbm/h</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.00</td>
<td>-3.9</td>
<td>-6.4</td>
</tr>
<tr>
<td>100</td>
<td>1570</td>
<td>1588</td>
<td>-1.130%</td>
<td>1585</td>
</tr>
<tr>
<td>500</td>
<td>7935</td>
<td>7940</td>
<td>-0.060%</td>
<td>7942</td>
</tr>
<tr>
<td>1000</td>
<td>15918</td>
<td>15880</td>
<td>0.249%</td>
<td>15888</td>
</tr>
<tr>
<td>2000</td>
<td>31767</td>
<td>31760</td>
<td>0.022%</td>
<td>31780</td>
</tr>
</tbody>
</table>

Linear Curve Fit Coefficients (y = mx + b where y = Measured Flow and x = Output)

<table>
<thead>
<tr>
<th>m</th>
<th>15.88</th>
<th>15.892</th>
<th>15.8927</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>0</td>
<td>-3.9</td>
<td>-6.4</td>
</tr>
</tbody>
</table>

**B2.7 Calculate the Bias and Precision Errors for Each Parameter.** The results from the previously defined elements are now summarized at each of the four calibrated flow rates in Table B-9. The summing of the terms is by the Root-Sum-Square method because a 95% confidence level is desired.
### TABLE B-9 Bias and Precision Error Summary

<table>
<thead>
<tr>
<th>Flow Rate (lbm/h)</th>
<th>% BIAS ERRORS</th>
<th>PRECISION ERRORS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calibration Facility Transmitter &amp;/or Application Summed Errors</td>
<td>Calibration Facility Transmitter &amp;/or Application Summed Errors</td>
</tr>
<tr>
<td></td>
<td>#1  #2         #1  #2          #1  #2</td>
<td></td>
</tr>
<tr>
<td>1588</td>
<td>0.035%       -1.130%   1.131% 0.015% 0</td>
<td>0.015%</td>
</tr>
<tr>
<td>7940</td>
<td>0.035%       -0.060%   0.069% 0.015% 0</td>
<td>0.015%</td>
</tr>
<tr>
<td>15880</td>
<td>0.035%       0.249%    0.252% 0.015% 0</td>
<td>0.015%</td>
</tr>
<tr>
<td>31760</td>
<td>0.035%       0.022%    0.042% 0.015% 0</td>
<td>0.015%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data Acquisition</th>
<th>#1   #2</th>
<th>#1   #2</th>
<th>#1   #2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.01%</td>
<td>0.012%</td>
<td>0.016%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data Reduction</th>
<th>#1</th>
<th>#2</th>
<th>0.813%</th>
<th>0.813%</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.067%</td>
<td>0.067%</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.203%</td>
<td>0.203%</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.044%</td>
<td>0.044%</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

#### B2.8 Propagate the Bias and Precision Errors

**B2.8.1** The bias and precision errors of the independent parameters are propagated separately all the way to the final test result. The individual terms are now summed together again by the Root-Sum-Square method because a 95% confidence level is desired as shown in the second and third columns of Table B-9.

**B2.8.2** Error propagation is done, according to the functional relationship of B2.5 above, via a Taylor series. This requires a calculation of the sensitivity factors, either by differentiation or by computer perturbation. Because the mass flow rate is the only independent parameter, this step is not required. The values determined in the last step are the ones that will be used in the next step to calculate uncertainty.

#### B2.9 Calculate Uncertainty

Select $U_{ADD}$ or $U_{RSS}$ as models for combining the bias and precision errors of the test result and obtain the uncertainty. $U_{RSS}$ will be used because a 95% confidence level is desired. The value of $t$ for the Student’s $t$ function is taken to be 2.00 because the degrees of freedom for the meter calibration is greater than 30. The final estimate is shown in the fourth column of Table B-10.
TABLE B-10 Propagation and Final Uncertainty Estimate

<table>
<thead>
<tr>
<th>Flow Rate (lbm/h)</th>
<th>PROPAGATED ERRORS</th>
<th>95% Confidence Uncertainty Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BIAS</td>
<td>PRECISION</td>
</tr>
<tr>
<td>1588</td>
<td>1.393%</td>
<td>0.015%</td>
</tr>
<tr>
<td>7940</td>
<td>0.098%</td>
<td>0.015%</td>
</tr>
<tr>
<td>15880</td>
<td>0.324%</td>
<td>0.015%</td>
</tr>
<tr>
<td>31760</td>
<td>0.063%</td>
<td>0.015%</td>
</tr>
</tbody>
</table>

B2.10 Report

B2.10.1 The report summary shall contain the nominal level of the test result, bias limit, precision index, degrees of freedom, and uncertainty of the test result, stating the model used.

B2.10.2 The report shall include a table of the elemental errors included in the uncertainty analysis along with the bias limit, precision index, and degrees of freedom of each parameter. The report for this analysis would include the average value determined per Section 5.2.1 at the actual test condition and the appropriate values from Tables B-4 and B-5.
INFORMATIVE ANNEX C: AN UNCERTAINTY ANALYSIS EXAMPLE FOR A DIFFERENTIAL PRESSURE FLOWMETER

A stainless steel ASME long-radius stainless-steel flow nozzle, in accordance with ANSI/ASME PTC 19.5, is installed in a laboratory test facility that will be used to measure superheated steam flow at a nominal pressure of 380 kPa (55 psi) and 160 °C (320 °F). Determine if the measurement system can meet an uncertainty of ± 0.75% at a 95% confidence level for mass flow.

Follow the step-by-step procedure outlined in Section 9 of ASME PTC 19.1, to estimate the uncertainty. Note that, in general, using a commercial equation solver software significantly reduces the time and effort required to complete an uncertainty analysis.

C1. Identify experimental goals and acceptable accuracy.

The initial problem statement defines the main objective of having an uncertainty level of ± 0.75% at a 95% confidence level. This statement does not specify if the uncertainty ± 0.75% refers to the full-scale reading of the meter or of the actual reading. It is assumed that the uncertainty will be based on the actual reading.

C2. Identify the important variables and appropriate relationships.

The equation from ASME PTC 19.5 that is used to calculate gas mass flow rate is given in Section 7.4 as Equation 7-5 in SI units or Equation 7-6 in I-P units where subscript 1 refers to the flowmeter inlet conditions and subscript 2 refers to the flowmeter throat.

\[
\dot{m} = C \varepsilon \left( \frac{\pi}{4} \right) d^2 \sqrt{\frac{2 \rho_1 (\Delta p)}{(1 - \beta^4)}} \tag{7-5}
\]

where

\[
\dot{m} = \text{gas mass flow rate, kg/s}
\]

\[
C = \text{discharge coefficient, dimensionless}
\]

\[
\varepsilon = \text{expansibility factor, dimensionless}
\]

\[
d = \text{throat diameter, m}
\]

\[
\rho_1 = \text{gas density, kg/m}^3
\]

\[
\Delta p = \text{absolute pressure, Pa}
\]

\[
\beta = d/D, \text{ dimensionless}
\]

\[
\dot{m} = C \varepsilon \left( \frac{\pi}{4} \right) d^2 \sqrt{\frac{2 g_c \rho_1 (\Delta p)}{(1 - \beta^4)}} \tag{7-6}
\]

where

\[
\dot{m} = \text{gas mass flow rate, lb m/s}
\]

\[
C = \text{discharge coefficient, dimensionless}
\]

\[
\varepsilon = \text{expansibility factor, dimensionless}
\]

\[
g_c = \text{gravitational constant, 32.174 (lb m/ft)/(lb ft/s}^2\)
\]

\[
d = \text{throat diameter, ft}
\]

\[
\rho_1 = \text{gas density, lb m/ft}^3
\]

\[
\Delta p = \text{absolute pressure, lb ft}^2
\]
\[ \beta = \frac{d}{D}, \text{ dimensionless} \]

The dimensionless expansibility factor, \( \varepsilon \), shall be obtained from Equation 7-8.

\[
\varepsilon = \left[ \frac{2}{\gamma} \left( \frac{\gamma - 1}{\gamma} \right) \left( \frac{1 - r/r^*}{1 - r} \right) \left( \frac{1 - \beta^4}{1 - \beta^4 + \beta^2 r^*} \right) \right]^{1/2}
\]  

(7-8)

where

\( \varepsilon = \text{expansibility factor, dimensionless} \)

\( r = \text{pressure ratio } p_2/p_1, \text{ dimensionless} \)

\( \gamma = \text{ratio of specific heats, dimensionless} \)

\( \beta = \text{diameter ratio } (d/D), \text{ dimensionless} \)

C3. Establish the quantities that must be measured and their expected range of variation.

The independent parameters, their nominal values, and expected range and variations are listed in the first five columns of Table C-1.

**Table C-1. Independent Parameters Descriptions**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Independent Parameter</th>
<th>Units</th>
<th>Nominal Value</th>
<th>Expected Range</th>
<th>Initial Selected Sensors/Instrument</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>Nozzle Throat Diameter</td>
<td>mm (inches)</td>
<td>20.32 (0.8)</td>
<td>10 to 30 (0.394 to 1.181)</td>
<td>Inside Diameter Micrometer</td>
</tr>
<tr>
<td>D</td>
<td>Pipe Diameter</td>
<td>mm (inches)</td>
<td>49.276 (1.94)</td>
<td>48.768 to 49.530 (1.920 to 2.00)</td>
<td>Inside Diameter Micrometer</td>
</tr>
<tr>
<td>t₁</td>
<td>Inlet Gas Temperature</td>
<td>°C (°F)</td>
<td>160 (320)</td>
<td>140 to 200 (284 to 392)</td>
<td>Type T thermocouple</td>
</tr>
<tr>
<td>p₁</td>
<td>Absolute Inlet Gas Pressure</td>
<td>kPa (PSIA)</td>
<td>379.2 (55)</td>
<td>275 to 825 (40 to 120)</td>
<td>Strain Gage Pressure Transducer</td>
</tr>
<tr>
<td>( \Delta P )</td>
<td>Nozzle Pressure Drop</td>
<td>kPa (PSIA)</td>
<td>68.95 (10)</td>
<td>34.5 to 172.4 (5 to 25)</td>
<td>Strain Gage Differential Pressure Transducer</td>
</tr>
<tr>
<td>C</td>
<td>Discharge Coefficient</td>
<td>none</td>
<td>0.995</td>
<td>0 to 1</td>
<td>Calibration from an ISO 17025 Certified Lab</td>
</tr>
</tbody>
</table>

Table C-2 shows relationships of the calculated parameters.
Table C-2. Relationships of the Calculated Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Dependent Parameter</th>
<th>Functional Relation</th>
<th>Functional Calculation</th>
<th>Nominal Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Nozzle Throat Area</td>
<td>$f(d)$</td>
<td>$\pi \frac{d^2}{4}$</td>
<td>0.03243</td>
<td>m² (ft²)</td>
</tr>
<tr>
<td>β</td>
<td>Nozzle Ratio</td>
<td>$f(d,D)$</td>
<td>$\frac{d}{D}$</td>
<td>0.4123</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$p_2$</td>
<td>Absolute Throat Pressure</td>
<td>$f(p_1,\Delta P)$</td>
<td>$p_1 - \Delta P$</td>
<td>310.2</td>
<td>kPa (psia)</td>
</tr>
<tr>
<td>r</td>
<td>Nozzle Pressure Ratio</td>
<td>$f(p_1,\Delta P)$</td>
<td>$\frac{p_2}{p_1}$</td>
<td>0.8182</td>
<td>dimensionless</td>
</tr>
<tr>
<td>γ</td>
<td>Ratio of the Specific Heats</td>
<td>$f(p_1,t_1)$</td>
<td>$\frac{c_p}{c_v}$</td>
<td>1.354</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Constant Pressure Specific Heat</td>
<td>$f(p_1,t_1)$</td>
<td>Property Data</td>
<td>2.1766</td>
<td>kJ/(kg·°C)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.5199)</td>
<td>(BTU/(lbm·°F))</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Constant Volume Specific Heat</td>
<td>$f(p_1,t_1)$</td>
<td>Property Data</td>
<td>1.6088</td>
<td>kJ/(kg·°C)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.3843)</td>
<td>(BTU/(lbm·°F))</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Inlet Gas Density</td>
<td>$f(p_1,t_1)$</td>
<td>Property Data</td>
<td>1.9559</td>
<td>kg/m³</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.1221)</td>
<td>(lbm/ft³)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Expansibility Factor</td>
<td>$f(p_1,t_1,\Delta P,d,D)$</td>
<td>Property Data</td>
<td>0.9719</td>
<td>Dimensionless</td>
</tr>
</tbody>
</table>
C4. Tentatively select sensors/instrumentation appropriate for the task. These also are listed in the last column of Table C-1.

C5. Document uncertainty of each measured variable. Individual sources of errors are detailed in Table C-3.

### Table C-3. Uncertainty of Each Measured Parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Error Origin</th>
<th>Calibration Error</th>
<th>Data Acquisition Error</th>
<th>Data Reduction Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bias Index</td>
<td>Precision Index</td>
<td>Bias Index</td>
</tr>
<tr>
<td>d</td>
<td>20.32 (0.8)</td>
<td>See Note 1 below</td>
<td>0.0305 (0.0012)</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>49.276 (1.94)</td>
<td>See Note 1 below</td>
<td>0.0508 (0.002)</td>
<td></td>
</tr>
<tr>
<td>t1</td>
<td>160 (320)</td>
<td>Calibration</td>
<td>0.194 (0.35)</td>
<td>0.028 (0.05)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ice Reference Junction</td>
<td>0.133 (0.24)</td>
<td>0.056 (0.1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Data Acquisition</td>
<td>0.167 (0.30)</td>
<td>0.044 (0.08)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Data Reduction/ Curve Fit</td>
<td></td>
<td>0.061 (0.11)</td>
</tr>
<tr>
<td>p1</td>
<td>379.2 (55)</td>
<td>Excitation Voltage</td>
<td>0.0010 (0.00015)</td>
<td>0.0007 (0.0001)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Signal Conditioning</td>
<td>0.0172 (0.0025)</td>
<td>0.0062 (0.0009)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calibration</td>
<td>4.1369 (0.6)</td>
<td>0.0041 (0.0006)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Data Acquisition</td>
<td></td>
<td>0.0276 (0.004)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Data Reduction/ Curve Fit</td>
<td></td>
<td>0.0083 (0.0012)</td>
</tr>
<tr>
<td>ΔP</td>
<td>68.95 (10)</td>
<td>Excitation Voltage</td>
<td>0.0103 (0.0015)</td>
<td>0.0014 (0.0002)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Signal Conditioning</td>
<td>0.0758 (0.011)</td>
<td>0.0048 (0.0007)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calibration</td>
<td>0.4137 (0.060)</td>
<td>0.0138 (0.002)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Data Acquisition</td>
<td></td>
<td>0.0434 (0.0063)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Data Reduction/ Curve Fit</td>
<td></td>
<td>0.1034 (0.015)</td>
</tr>
<tr>
<td>C</td>
<td>0.995</td>
<td>Uncertainty from a independent calibration</td>
<td>0.0025</td>
<td></td>
</tr>
</tbody>
</table>
Note 1 – Because this meter was calibrated, error effects in the throat and pipe diameters are eliminated provided the same dimensions used during the calibration are also used in the test calculations and they have not changed due to damage such as erosion. The Calibration Bias Index is included to allow calculation of $\beta$ and $\epsilon$.

C6. Perform a preliminary uncertainty analysis.
In order to perform the preliminary uncertainty analysis, the functional relationship between the independent measurement parameters and the test results must be determined. The two main equations stated in Step 2 of this annex define the overall measurement relationships. Six other dependent parameters are related to the parameters in Table C-1 as shown in Table C-3.

C6.1 Calculate the Bias and Precision Errors for Each Parameter.
The estimates from the previously defined elements in Table D-3 are now summarized for each elemental term using the Root-Sum-Square method since 95% confidence is desired. The degrees of freedom for each parameter is greater than 30, so the Student’s $t$ function will be 2.

For $t_1$:
SI:
\[
B_{t_1} = [(0.194)^2 + (0.133)^2 + (0.167)^2 + (0.061)^2]^{1/2} \\
= [(0.0378) + (0.0178) + (0.0278) + (0.0037)]^{1/2} = 0.2951
\]
\[
S_{t_1} = [(0.028)^2 + (0.056)^2 + (0.044)^2 + (0.009)^2]^{1/2} \\
= [(0.00077) + (0.00309) + (0.00198) + (0.00009)]^{1/2} = 0.0770
\]
\[
U_{t_1} = [(B_{t_1})^2 + (2 \times S_{t_1})^2]^{1/2} = [(0.2951)^2 + (2 \times 0.0770)^2]^{1/2} = 0.333
\]
IP:
\[
B_{t_1} = [(0.35)^2 + (0.24)^2 + (0.3)^2 + (0.11)^2]^{1/2} \\
= [(0.1225) + (0.0576) + (0.09) + (0.012)]^{1/2} = 0.531
\]
\[
S_{t_1} = [(0.05)^2 + (0.1)^2 + (0.08)^2 + (0.017)^2]^{1/2} \\
= [(0.0025) + (0.01) + (0.0064) + (0.00029)]^{1/2} = 0.139
\]
\[
U_{t_1} = [(B_{t_1})^2 + (2 \times S_{t_1})^2]^{1/2} = [(0.531)^2 + (2 \times 0.139)^2]^{1/2} = 0.599
\]

For $p_1$:
SI:
\[
B_{p_1} = [(0.001)^2 + (0.0172)^2 + (4.1369)^2 + (0.0276)^2 + (0.0083)^2]^{1/2} \\
= \left[\left((1.07) + (297.1) + (17113564) + (760.6) + (68.45)\right) \times 10^{-6}\right]^{1/2} = 4.137
\]
\[
S_{p_1} = [(0.0007)^2 + (0.0062)^2 + (0.0041)^2 + (0.0345)^2]^{1/2} \\
= \left[\left((0.47) + (38.5) + (171.11) + (1188)\right) \times 10^{-6}\right]^{1/2} = 0.0353
\]
\[
U_{p_1} = [(B_{p_1})^2 + (2 \times S_{p_1})^2]^{1/2} = [(4.137)^2 + (2 \times 0.0352)^2]^{1/2} = 4.138
\]
IP:
\[
B_{p_1} = [(0.00015)^2 + (0.0025)^2 + (0.6)^2 + (0.004)^2 + (0.0012)^2]^{1/2} \\
= \left[\left((0.0225) + (6.25) + (360000) + (16.0) + (1.44)\right) \times 10^{-6}\right]^{1/2} = 0.600
\]
\[
S_{p_1} = [(0.0001)^2 + (0.0009)^2 + (0.0006)^2 + (0.005)^2]^{1/2} \\
= \left[\left((0.01) + (0.810) + (0.360) + (25.0)\right) \times 10^{-6}\right]^{1/2} = 0.00512
\]
\[
U_{p_1} = [(B_{p_1})^2 + (2 \times S_{p_1})^2]^{1/2} = [(0.600)^2 + (2 \times 0.00512)^2]^{1/2} = 0.600
\]
For $\Delta P$:

**SI:**

\[
\begin{align*}
B_{\Delta P} &= [(0.0103)^2 + (0.0758)^2 + (0.4137)^2 + (0.0434)^2 + (0.1034)^2]^{1/2} \\
&= \left[\left((0.107) + (5.752) + (171.1) + (1.887) + (10.7)\right) \times 10^{-3}\right]^{1/2} = 0.4354
\end{align*}
\]

\[
\begin{align*}
S_{\Delta P} &= [(0.0014)^2 + (0.0048)^2 + (0.0138)^2 + (0.1655)^2]^{1/2} \\
&= \left[\left((1.902) + (23.29) + (190.2) + (27380)\right) \times 10^{-6}\right]^{1/2} = 0.1661
\end{align*}
\]

\[
\begin{align*}
U_{\Delta P} &= [(B_{\Delta P})^2 + (2 \times S_{\Delta P})^2]^{1/2} = [(0.4354)^2 + (2 \times 0.1661)^2]^{1/2} = 0.548
\end{align*}
\]

**IP:**

\[
\begin{align*}
B_{\Delta P} &= [(0.0015)^2 + (0.011)^2 + (0.060)^2 + (0.0063)^2 + (0.015)^2]^{1/2} \\
&= \left[\left((2.25) + (121) + (3600) + (39.7) + (225)\right) \times 10^{-6}\right]^{1/2} = 0.06315
\end{align*}
\]

\[
\begin{align*}
S_{\Delta P} &= [(0.0002)^2 + (0.0007)^2 + (0.02)^2 + (0.024)^2]^{1/2} \\
&= \left[\left((0.04) + (0.49) + (4.00) + (576.0)\right) \times 10^{-6}\right]^{1/2} = 0.0241
\end{align*}
\]

\[
\begin{align*}
U_{\Delta P} &= [(B_{\Delta P})^2 + (2 \times S_{\Delta P})^2]^{1/2} = [(0.06315)^2 + (2 \times 0.0241)^2]^{1/2} = 0.0794
\end{align*}
\]

For C:

\[
\begin{align*}
B_C &= [(0.0025)^2]^{1/2} = [(6.25) \times 10^{-6}]^{1/2} = 0.0025
\end{align*}
\]

\[
\begin{align*}
S_C &= 0.0
\end{align*}
\]

\[
\begin{align*}
U_C &= [(B_C)^2 + (2 \times S_C)^2]^{1/2} = [(0.0025)^2 + (2 \times 0.0)^2]^{1/2} = 0.0025
\end{align*}
\]

Note: The uncertainty is calculated at this time for $t_1$, $p_1$, and $\Delta P$ so that they may be used in estimating errors in $\rho$ and $\epsilon$ in the next step.

**C6.2 Propagate the Bias and Precision Errors.** The individual parameter errors are propagated into the flow rate according to a Taylor series expansion. The relative bias limit for the mass flow equation is:

\[
\frac{B_m}{m} = \left[\left(1 \times \frac{B_C}{C}\right)^2 + \left(\frac{2}{1 - \beta^*} \times \frac{B_d}{d}\right)^2 + \left(\frac{2}{1 - \beta^*} \times \frac{B_D}{D}\right)^2 + \left(\frac{1}{2} \times \frac{B_{\Delta P}}{\rho}\right)^2 + \left(\frac{1}{2} \times \frac{B_{\epsilon}}{\rho}\right)\right]^{1/2}
\]

Similarly the relative precision index for the flow rate can be written as:

\[
\frac{S_m}{m} = \left[\left(1 \times \frac{S_C}{C}\right)^2 + \left(\frac{2}{1 - \beta^*} \times \frac{S_d}{d}\right)^2 + \left(\frac{2}{1 - \beta^*} \times \frac{S_D}{D}\right)^2 + \left(\frac{1}{2} \times \frac{S_{\Delta P}}{\rho}\right)^2 + \left(\frac{1}{2} \times \frac{S_{\epsilon}}{\rho}\right)\right]^{1/2}
\]

Since $d$ and $D$ are already accounted for in the C term, the second and third term of the above equations are zero. The density term, $\rho$, is a function of $t_1$ and $p_1$, and the variation is estimated by observing the change in density properties at the extremes of expected $t_1$ and $p_1$ as shown in Table C-4. The expansion factor term, $\epsilon$, is a function of $t_1$, $p_1$, and $\Delta P$, and the variation is similarly evaluated as shown in Table C-5.
Table C-4. Calculated Density at Minimum, Nominal, and Maximum

<table>
<thead>
<tr>
<th></th>
<th>( t_1 )</th>
<th>( p_1 )</th>
<th>( \rho )</th>
<th>( c_p )</th>
<th>( c_v )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal SI (IP)</td>
<td>160.0 (320)</td>
<td>379.2 (55)</td>
<td>1.9559 (0.1221)</td>
<td>2.1766 (0.5158)</td>
<td>1.6088 (0.3825)</td>
<td>1.353</td>
</tr>
<tr>
<td>Max Value SI (IP)</td>
<td>160.33 (320.6)</td>
<td>383.3 (55.61)</td>
<td>1.9794 (0.1236)</td>
<td>2.1807 (0.5184)</td>
<td>1.6111 (0.3828)</td>
<td>1.354</td>
</tr>
<tr>
<td>Min Value SI (IP)</td>
<td>159.67 (319.4)</td>
<td>375.1 (54.38)</td>
<td>1.9325 (0.1206)</td>
<td>2.1724 (0.5174)</td>
<td>1.6065 (0.3822)</td>
<td>1.352</td>
</tr>
</tbody>
</table>

Using the maximum and minimum values to define the variation,
SI:  
\[ \rho = 1.9559 \pm 0.0235 \quad \gamma = 1.353 \pm 0.001 \]

IP:  
\[ \rho = 0.1221 \pm 0.0015 \quad \gamma = 1.353 \pm 0.001 \]

\( P_2 \) variation is estimated based on the combined uncertainty of \( P_1 \) and \( \Delta P \).

\[ U_{P_2} = \left( (U_{P_1})^2 + (U_{\Delta P})^2 \right)^{1/2} \]

SI:  
\[ = \left( (4.138)^2 + (0.548)^2 \right)^{1/2} = 4.174 \]

IP:  
\[ = \left( (0.600)^2 + (0.0794)^2 \right)^{1/2} = 0.605 \]

The micrometer accuracy for the throat and pipe diameter measurements is ± 0.127 mm.

In determining the maximum and minimum values of \( \varepsilon \), the eight combinations of \( r \), \( \gamma \), and \( \beta \) were calculated and the maximum value of \( \varepsilon \) occurred when \( r \) and \( \gamma \) were at their minimum values and \( \beta \) was at the maximum value, and the minimum value of \( \varepsilon \) occurred when \( r \) and \( \gamma \) were at their maximum values and \( \beta \) was at the minimum value.

Table C-5. Minimum, Nominal, and Maximum Values of \( \varepsilon \)

<table>
<thead>
<tr>
<th></th>
<th>( P_1 )</th>
<th>( \Delta P )</th>
<th>( P_3 )</th>
<th>( r )</th>
<th>( \gamma )</th>
<th>( \beta )</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal SI (IP)</td>
<td>379.2 (55.0)</td>
<td>68.95 (10.0)</td>
<td>310.2 (45.00)</td>
<td>0.8182</td>
<td>1.3529</td>
<td>0.4123</td>
<td>0.8906</td>
</tr>
<tr>
<td>Max Value SI (IP)</td>
<td>383.3 (55.59)</td>
<td>69.5 (10.08)</td>
<td>314.4 (45.60)</td>
<td>0.8202</td>
<td>1.3536</td>
<td>0.4127</td>
<td>0.8919</td>
</tr>
<tr>
<td>Min Value SI (IP)</td>
<td>375.1 (54.40)</td>
<td>68.4 (9.92)</td>
<td>306.0 (44.39)</td>
<td>0.8159</td>
<td>1.3523</td>
<td>0.4120</td>
<td>0.8891</td>
</tr>
</tbody>
</table>

Averaging the maximum and minimum values,
\[ \varepsilon = 0.8906 \pm 0.0014 \]

Based on previous experience, the variations for \( \rho \) and \( \varepsilon \) will be assumed to be entirely bias errors.

1. The bias and precision errors of the independent parameters are propagated separately all the way to the final test result.
2. Error propagation is done, according to the functional relationship of (a)(4) above, via a Taylor series. This requires a calculation of the sensitivity factors, either by differentiation or by computer
perturbation. The individual terms are now summed together by the Root-Sum-Square method since a 95% confidence level is desired as shown below.

\[
\frac{B_m}{m} = \left[ \left( 1 \times \frac{B_c}{C} \right)^2 + \left( \frac{2}{1 - \beta^4} \times \frac{B_d}{d} \right)^2 + \left( \frac{2}{1 - \beta^4} \times \frac{B_p}{D} \right)^2 + \left( \frac{1}{2} \times \frac{B_\rho}{\rho} \right)^2 + \left( \frac{1}{2} \times \frac{B_{AP}}{\Delta P} \right)^2 + \left( 1 \times \frac{B_\epsilon}{\epsilon} \right)^2 \right]^{1/2}
\]

SI:

\[
\frac{B_m}{m} = \left[ \left( 0.0025 \times 0.9950 \right)^2 + (0)^2 + (0)^2 + \left( \frac{1}{2} \times \frac{0.0235}{1.9559} \right)^2 + \left( \frac{1}{2} \times \frac{0.4354}{68.95} \right)^2 + \left( 1 \times \frac{0.0014}{0.8906} \right)^2 \right]^{1/2}
\]

\[
\frac{B_m}{m} = \left[ \left( (6.313) + (0) + (0) + (36.09) + (9.969) + (2.471) \times 10^{-6} \right) \right]^{1/2} = 0.00740
\]

\[
\frac{S_m}{m} = \left[ (0)^2 + (0)^2 + (0)^2 + (0)^2 + \left( \frac{1}{2} \times \frac{0.1661}{68.95} \right)^2 \right]^{1/2} = \left[ (0.001204)^2 \right]^{1/2} = 0.001204
\]

\[
U_w = \sqrt{\left( \frac{B_m^2}{m^2} + 2S_m^2 \right)} = \sqrt{0.00740^2 + 2 \times 0.00001204^2} = \sqrt{54.84 + 2.901} \times 10^{-6} = 0.0076 = 0.76% \text{; } P=95%
\]

IP:

\[
\frac{B_m}{m} = \left[ \left( 0.0025 \times 0.9950 \right)^2 + (0)^2 + (0)^2 + \left( \frac{1}{2} \times \frac{0.001468}{0.1221} \right)^2 + \left( \frac{1}{2} \times \frac{0.06315}{10} \right)^2 + \left( 1 \times \frac{0.0014}{0.8906} \right)^2 \right]^{1/2}
\]

\[
\frac{B_m}{m} = \left[ \left( (6.313) + (0) + (0) + (36.09) + (9.969) + (2.471) \times 10^{-6} \right) \right]^{1/2} = 0.00740
\]

\[
\frac{S_m}{m} = \left[ (0)^2 + (0)^2 + (0)^2 + (0)^2 + \left( \frac{1}{2} \times \frac{0.0241}{10} \right)^2 \right]^{1/2} = \left[ (0.001204)^2 \right]^{1/2} = 0.001204
\]

\[
U_m = \sqrt{\left( \frac{B_m^2}{m^2} + 2S_m^2 \right)} = \sqrt{0.00740^2 + 2 \times 0.00001204^2} = \sqrt{54.84 + 2.901} \times 10^{-6} = 0.0076 = 0.76% \text{; } P=95%
\]

C7. Study uncertainty results and reassess the ability of the measurement methods and instrumentation to meet acceptable accuracy.

The overall uncertainty is just over the 0.75% goal at 0.76%. The selected system should meet the intent of 0.75% uncertainty goal initially presented in the problem statement. To add a safety factor, the largest contribution to the bias uncertainty is the differential pressure measurement as it has a value of 9.969 E-06 in 6.2 above which equates to 0.78% of the pressure reading. If this measurement has its bias decreased by a factor of 2, the resulting overall uncertainty is decreased to 0.71% which is under the 0.75% goal.

C8. Install selected instrumentation in accord with relevant standards or best practices.

All the initially specified measurement system will be used with the exception of the differential pressure measurement that will use a sensor that is twice as accurate as initially specified.

C9. Perform initial verification of data quality.

A data acquisition system is assembled and a data reduction program was developed and verified to match hand calculations for the flow measurement.

C10. Collect experimental data subject to ongoing quality control criteria.
Three rounds of data were taken as required by the test plan.

C11. Accomplish data reduction and analysis.
A summary of all the data and calculations for this test run are shown in Table C-6.

Table C6. Summary of Calculations

<table>
<thead>
<tr>
<th>Fixed Nozzle Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>C</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculated Fixed Nozzle Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>β</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data Round Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round Number</td>
</tr>
<tr>
<td>t_1</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>p_1</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Δp</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculated Parameters for each data round</th>
</tr>
</thead>
<tbody>
<tr>
<td>p_2</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>R</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>c_p</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>c_v</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>γ</td>
</tr>
<tr>
<td>P</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>ε</td>
</tr>
<tr>
<td>m</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

C12. Perform final uncertainty analysis.
Since the actual test parameters are very close to the initially estimated values, the initial analysis should still be valid.

The final report would include all the data from Table D-6 and sufficient information to accurately define the testing installation. This would include the associated supporting documentation such as calibration reports on the pressure and temperature instrumentation, data acquisition equipment, technician responsible
for the data, test date, time and location. The final value for flow rate would be described as:

- **SI:** $\dot{m} = 0.4801 \pm 0.0034 \text{ kg/s}; \text{ P 95\%}$
- **IP:** $\dot{m} = 1.058 \pm 0.0008 \text{ lbm/s}; \text{ P 95\%}$

The $\pm 0.034$ is found by taking the average value, 0.4801, and multiplying by the uncertainty percentage determined in Step C7, 0.709%.
INFORMATIVE ANNEX D: FLOWMETER ACCURACY COMPARISONS

Table D-1 provides examples of the accuracy of commercial refrigerant flowmeters for comparison purposes.

TABLE D-1 Examples of commercial gas flowmeter accuracies for comparison purposes

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coriolis Flowmeters</td>
<td>±0.1%</td>
</tr>
<tr>
<td>Thermal Flowmeters</td>
<td>±1%</td>
</tr>
<tr>
<td>Volume-Displacement Flowmeters</td>
<td>±0.5%</td>
</tr>
<tr>
<td>Orifice Meters</td>
<td>±0.5%</td>
</tr>
<tr>
<td>Flow Nozzles</td>
<td>±0.5%</td>
</tr>
<tr>
<td>Venturi Tubes</td>
<td>±0.5%</td>
</tr>
<tr>
<td>Turbine Flowmeters</td>
<td>±0.25%</td>
</tr>
<tr>
<td>Variable-Area Flowmeters</td>
<td>±2%</td>
</tr>
<tr>
<td>Ultrasonic Flowmeters</td>
<td>±1%</td>
</tr>
<tr>
<td>Vortex-Shedding Flowmeters</td>
<td>±0.75%</td>
</tr>
<tr>
<td>Drag-Force Flowmeters</td>
<td>±2%</td>
</tr>
</tbody>
</table>